The effects of costly consumer search on mergers and cartels

Petrikaité, Vaiva

The Effects of Costly Consumer Search on Mergers and Cartels

Vaiva Petrikaitė

Publisher: University of Groningen, Groningen, The Netherlands
Printer: Ipskamp Drukkers B.V.
ISBN: 978-90-367-5447-7
eISBN: 978-90-367-5446-0

This thesis was typeset in LaTeX using Ward Romp's much appreciated style file.

The Effects of Costly Consumer Search on Mergers and Cartels

Proefschrift

ter verkrijging van het doctoraat in de
Economie en Bedrijfskunde
aan de Rijksuniversiteit Groningen
op gezag van de
Rector Magnificus, dr. E. Sterken,
in het openbaar te verdedigen op
donderdag 24 mei 2012
om 12:45 uur

door

Vaiva Petrikaitė

geboren op 10 december 1981
te Utena, Litouwen

Promotor	:	Prof. dr. J.L. Moraga-González
Beoordelingscommissie	:	Prof. dr. J. Boone
		Prof. dr. M.C.W. Janssen
		Prof. dr. R. Renault

ISBN: 978-90-367-5447-7
eISBN: 978-90-367-5446-0

Acknowledgments

Long time ago one person told me that a human life is like a travel by train. A man keeps changing trains and carriages during this travel. He greets new travel companions after a change and says goodbye to his long-standing friends before he leaves for the next train. Every piece of the journey, every travel companion gives the man experience, advice and some important lessons which have to learn for the future.

This thesis marks one of my life travel milestones. It would not have been reached without the people who traveled with me up to this train station. The most sincere and the warmest *'thank you'* I want to say to my supervisor José Luis Moraga-González for his never ending patience, for the time that he spent in explaining me the laws of economics, academic life and writing, for sharing his invaluable experience, knowledge and research secrets and for the seed of a healthy scientific ambition.

I want to express my sincere gratitude to the European Commission for Marie Curie Actions which gave me the opportunity to pursue a PhD degree, to enrich my knowledge and to make valuable connections with other European researchers. I am also thankful to the reading committee Jan Boone, Maarten Janssen and Régis Renault for the evaluation of my thesis and useful comments. I cannot help sending sincere thanks to SOM Bureau for their assistance in handling daily issues. I am also grateful to Marco Haan, Umed Temurshoev, Federico Crudu, Marielle Non, Zsolt Sándor and other colleagues from the Faculty of Economics for discussions, advice, sincere opinions and suggestions that helped me to reach this stage. A very warm thanks I send to my paranymphs who are wonderful colleagues and trustworthy friends. The last but not the least *thank you* I give to Ahti Heinla, Priit Kasesalu and Jaan Tallinn for their wonderful invention *Skype* which has been the most trustable and loyal communication intermediary.

In the last four years I had a lot of wonderful people next to me, who made me see things in a different light, supported in hard moments and gave me an opportunity to do and fix my mistakes. I remember everyone with gratitude and pleasure. Thank you all for being nearby and let me wish you all a lot of success in catching the right train.

Contents

Chapter 1

Introduction

1.1 Brief overview of the thesis

Nowadays regulatory bodies and competition policy makers borrow heavily from the findings of theoretical and empirical economic models.[1] However, the European Commission (the EC) warns that this has to be done cautiously because by '*their very nature, economic models and arguments are based on simplifications of reality*'.[2] Some of the simplifications, e.g. rationality of economic agents, the shape of demand and supply functions, may be crucial for the outcomes of the models. Consequently, the predictions drawn from the findings in economics models may differ significantly from the observations in reality.

A typical assumption in economic models is that all the consumers are fully informed about all firms' offers and can pick the one providing them with the highest utility. However, in many real markets information about available offers is costly to obtain. In this type of markets the optimal consideration set and the ultimate choice of a consumer may differ significantly from the one in a standard model. Since consumer decisions shape the price strategies of sellers, the results of economic models with fully and partially informed consumers may be quite different. This thesis contributes to this line of reasoning by introducing *consumer search costs*[3] in standard models of collusion and mergers. Our analysis reveals that the outcome of a merger (Chapters 2 and 3) and the incentives to collude (Chapter 4) depend

[1] Röller (2006) observes that now the question is '*what kind of economics and especially how the economic analysis is used*' instead of how much economics is needed.

[2] DGCompetition (2010)

[3] See section 1.2 for the definition of consumer search costs.

heavily on the magnitude of search costs. The relevance of these results rests on whether search costs are actually large or not. This thesis also contributes to the measurement of search costs by developing an empirical consumer search model for differentiated products (Chapter 5).

The incentives to merge and the welfare implications of mergers continue to be important topics of study in Industrial Organization (see Shapiro, 1989; Whinston, 2007; Porter, 1991, Ivaldi, Jullien, Rey, Seabright, and Tirole, 2003a). The seminal works of Salant, Switzer, and Reynolds (1983) and Deneckere and Davidson (1985) show, respectively, that firms do not have incentives to merge in Cournot markets for homogeneous products, while they do have in Bertrand markets for horizontally differentiated products, with corresponding negative welfare consequences. In Chapter 2 of this thesis, using a model of consumer search for differentiated products and Bertrand competition, we demonstrate that a merger paradox can also arise if consumer search costs are sufficiently high. The basic insight goes as follows. The standard internalization of pricing externalities makes the price of a merger to differ from the price of non-merged firms. This difference in prices, along with the fact that consumers face positive search cost, imply a particular optimal search order, namely, consumers start searching from non-merged sellers and visit the shops of a merger only after all the non-merged firms are visited. This search order has a strong negative effect on the quantity of the merger. Hence, the merger earns less than in a pre-merger market, despite the higher post-merger price level.

The studies of Farrel and Shapiro (1990), Williamson (1968), Perry and Porter (1985), Lommerud and Sørgard (1997) and Whinston (2007) reveal that a merger is welfare improving if it brings sufficient cost efficiencies or increases product variety. In Chapter 3 of this thesis we show that mergers have the potential to be welfare improving without any supply side efficiencies when consumer search is sufficiently costly. The mechanism is as follows. Suppose the firms that merge start selling all the varieties of the parent firms so that they effectively reduce the consumer costs of searching various alternatives. We show that when search costs are sufficiently high mergers that result in multi-product shops lead to consumer surplus increase even if the post merger prices are above the pre-merger equilibrium price.

Collusion has also been a theme that has received a lot of attention in industrial organization (see Jacquemin and Slade, 1989; Ivaldi et al., 2003b). Part of that attention has been on how market transparency affects the stability of collusion. Borrowing on that body of work, market transparency is assumed to be beneficial for colluding sellers by competition authorities.[4] The studies of Green and Porter (1984) suggest that collusion stability increases if the monitoring of cartel members is easier. Chapter 4 of this thesis adds to this line of work by addressing the relation between cartel stability and market transparency from a different angle. Rather than focusing on how firms can monitor one another, we study the incentives to collude when consumers face search costs. Our analysis reveals that market transparency from the consumer point of view counters with market transparency from the seller point of view. In other words, a cartel becomes more stable if the search costs go up.

The previously mentioned chapters add to the theoretical body of work pointing to the fact that search costs have an important influence in economic activity (see Stigler, 1961; Burdett and Judd, 1983; Stahl, 1989; Wolinsky, 1986; Anderson and Renault, 1999). This body of work has recently led to a new area in empirical economics dedicated to the estimation of search costs (see Hong and Shum, 2006; Hortaçsu and Syverson, 2004; Kim, Albuquerque, and Bronnenberg, 2010; Moraga-González and Wildenbeest, 2008). This thesis also contributes to this body of knowledge by developing in Chapter 5 an empirical model of sequential search for differentiated products. The model uses the well-known multinomial logit (MNL) demand model. The MNL model has attracted the attention of economists and competition policy practitioners because, *'despite [a] potentially large number of dimensions of consumer "types"[...], the resulting demand functions are entirely analytic, making analysis and estimation relatively straightforward.'*[5] As a result, there are many modifications and estimation techniques of this model in the economics literature. We modify the market-share-based MNL demand model by introducing sequential consumer search. We show that the estimated parameter values are strongly affected by the assumption how the consideration set of a consumer is formed. Costly search also effects the estimation routine.

Chapter 6 of this thesis is dedicated to summarize the main findings of this work as well as to describe some ideas for future research.

1.2 Literature on consumer search costs

There is lots of advice for consumers by competition authorities and other consumer rights protecting institutions. For instance, the Federal Trade Commission

[4] E.g. Federal Trade Commission (2010), section 7.2 'Evidence a Market is Vulnerable to Coordinated Conduct'

[5] Davis and Garcés (2010) Chapter 9 "Demand System Estimation"

(the FTC) suggests that a wise consumer should shop around, as the sale price is not always the "*best*" price.[6] Moreover, the shopper should read the adds carefully and notice such details as "*quantities limited*", or "*not available in all stores*". The EC even advices shoppers to check price differences among the EU member-states, as well as differences in the terms of any additional commercial guarantees like refund, repair, etc.[7]

The existence of this amount of consumer advice indicates that buyers often make their purchase decisions without collecting and analysing all necessary information. Such behaviour may look irrational at first sight. However, it needs not to be. Information gathering and its analysis require that buyers spend time and effort.[8] Hence, consumers must trade-off the wish to buy the good and(or) service that gives them the highest possible surplus against the costs of being informed. Consequently, some consumers deliberately choose to be uninformed or to be partially informed when they make their purchase decisions.

The fact that some consumers are not fully informed about available offers in a market has an impact on firms' pricing decisions. This issue has been broadly discussed in the academic literature. We start the review of the economic models on costly consumer search by considering the theoretical work. Afterwards we continue with the summary of the empirical economic models, which are used to estimate consumer search costs.

1.2.1 Theoretical models

The theoretical models that analyse the effect of consumer search costs on equilibrium prices and social welfare can be classified in two groups. The first group consists of the models that deal with homogenous product markets. The second group of models focuses on differentiated product markets. In all papers firms maximize their profits by choosing prices. Whether the equilibrium is in mixed or pure strategies depends on whether products are homogeneous or differentiated.

Stigler (1961) has already pointed out that firms selling homogeneous products will charge different prices if consumers search for prices at positive costs. When search is costly, consumers rarely compare all the market offers and so the observed minimum price may be higher than the minimum market price. Since the con-

[6] Some advice can be found on the website of FTC: *http://www.ftc.gov/bcp/consumer.shtm*.

[7] The European Consumer Centres Network *http://ec.europa.eu/consumers/ecc/index_en.htm*.

[8] For instance, FTC warns that a smart consumer must *'take time and travel cost into consideration'* while she is looking for the best price and quality match (FTC "Shopping Tips: Is That Deal for Real?" available at *http://www.ftc.gov/bcp/edu/pubs/consumer/alerts/alt081.shtm*.).

sumer will pay the minimum observed price, firms dare to take a risk and charge high prices with positive probabilities. Diamond (1971) addresses costly consumer search in a homogeneous product market where a consumer can observe the price of only one store per period at a positive search cost. The consumer chooses between taking the observed current offer or searching one more seller in the next period. Firms make their pricing decisions every period. Diamond (1971) shows that firms set their prices slightly below the sum of the price that a consumer would agree to pay in the subsequent period and the search cost. As a result, sellers charge the monopoly price and consumers do not search beyond the first visited shop in equilibrium. Varian (1980) has shown that the Diamond paradox fails to hold if there are some consumers who are perfectly informed about all prices. In his model a firm needs to find a balance between charging the monopoly price and serving only non-informed consumers and having the lowest price and selling many more units. Varian (1980) demonstrates that in that case there is a unique equilibrium price distribution.

There are two main types of search protocols in the theoretical consumer search models: *sequential* and *non-sequential* (*simultaneous*) consumer search. If a consumer firstly decides on the number of offers she wants to search and makes the purchasing decision only after all chosen alternatives are observed then the consumer is said to search non-sequentially. A consumer searches sequentially if she makes the decision whether to sample one more store or stop searching and accept the best observed offer after every visit to a shop.

No matter the search protocol employed by consumers, the price equilibrium in homogeneous product markets is in mixed strategies. Burdett and Judd (1983) have assumed that consumers search non-sequentially and have shown that there may be one and more dispersed price equilibria in their model. Janssen and Moraga-González (2004) introduced a fraction of zero search cost consumers in the model of Burdett and Judd (1983) and assumed a finite number of firms. There is no equilibrium in pure strategies in their model too, and the expected price does not decrease with the number of firms. Stahl (1989) studies a model where some consumers have zero search costs and others have positive search costs, but with sequential search rather than non-sequential. He also characterizes an equilibrium in mixed strategies that bridges between the monopoly price equilibrium and the competitive price equilibrium as the fraction of zero search cost consumers goes form zero to one. Janssen, Moraga-González, and Wildenbeest (2005) examined equilibrium properties in a slightly modified version of Stahl (1989). More particularly, they assume that the searching consumers do not get the first price quote for free. In their model the equilibrium is also in mixed strategies. Additionally, they show that when the search cost is hight, the searching consumers may decide to stay outside the market with strictly positive probability.

By contrast, horizontally differentiated product markets with costly consumer search typically exhibit equilibria in pure strategies. The equilibrium price(s) is(are) above marginal production costs because firms have some market power. Knauff (2006) analyses the equilibrium in a Hotelling model where a fraction of consumers are not informed about the prices. She has shown that *'an increase in market transparency*[9] *causes the equilibrium prices of both goods to decrease'*. Wolinsky (1986) demonstrates that the symmetric equilibrium price is above marginal production costs if consumer search is costly, even if the number of firms tends to infinity. Moreover, the analysis of Anderson and Renault (1999) reveals that the equilibrium price in a differentiated product market increases with search costs.

In all this work, a key result is that prices become higher as consumer search costs increase. Governmental institutions that protect consumer welfare take good account of this result. For instance according to a survey of the Irish National Consumer Agency (INCA), in 2008 British retailers charged Irish consumers 60% more than British customers.[10] The surveyed retailers argued they faced higher operating costs in Ireland than in Great Britain. However, Irish consumers would have benefited by buying via Internet from the stores in Great Britain even after taking into account currency exchange and delivery costs. The INCA survey in 2010 found that just over 7 in 10 consumers stated that they shopped for better prices.[11] As a result, the agency advises consumers to *'watch price labels'*, *'shop around'*, *'remember discount stores'*, etc.[12]

When gathering price information is costly for consumers, there is also a role for firm advertising. Robert and Stahl II (1993) have analysed price advertising in a homogeneous product market with sequentially searching consumers. Robert and Stahl II (1993) have shown that firms *'advertise "sale" prices more than high prices'*, but they advertise less if the search costs approach zero. Haan and Moraga-González (2011) have examined advertising incentives in a horizontally differentiated product market with sequential consumer search. In their model advertising attracts consumers' attention but does not reveal the details about firms' offers. Their anal-

[9] The fraction of informed consumers.
[10] INCA (2008)"Watch out for sterling price differences" available at *http://www.consumerconnect.ie*.
[11] ICNA (2011) "Consumer attitudes to shopping and pricing" available at *http://www.consumerconnect.ie*.
[12] See footnote 10.

ysis reveals that firms advertise more if the search cost goes up and the distribution function of match values is convex. However, the equilibrium price goes up if the search cost increases.

Recently, the search cost literature has also paid attention to the origin of search costs and has recognized that to some extent search costs are created by the firms themselves. In this regard, a relevant question is whether firms are interested in increasing or decreasing search costs. On the one hand, a seller that charges a high price can sell more if a customer has difficulties to compare its offer with the offers of other firms. As a result, firms may want to make their products, the descriptions of their price components, etc. less understandable for consumers. The practice of doing this is called *obfuscation*. On the other hand, higher search costs may reduce overall demand. Ellison and Wolitzky (2009) have modified the model of Stahl (1989) by introducing the possibility for firms to engage in obfuscation. They find that firms obfuscate in equilibrium. Moreover, if a firm charges a higher price then it obfuscates more. Wilson (2010) also looks at the market equilibrium in a sequential consumer search model with homogeneous products. In contrast to Ellison and Wolitzky (2009), he assumes that consumers can observe the obfuscation level of a firm before the visit. Wilson (2010) shows that there is no market equilibrium where firms choose not to obfuscate.[13]

Models of consumer search costs have also been prominent in the study of firm location, firm agglomeration and product differentiation. A firm that provides a consumer with the highest expected surplus has the highest chance to get into the consideration set of the consumer. A consumer gets higher expected surplus at a firm if it charges a lower price, offers more varieties or (and) is searched at a lower search cost than other sellers. Hence, firms may decide to settle near each other (join a cluster). If the firms enter a cluster then the search costs per clustered firm are less than the search costs per stand-alone seller. Then competition between clustered firms is stronger than between stand-alone shops. Prices and profits of sellers go down if competition becomes more intense. This has a negative effect on the incentives to cluster. Nevertheless, the results of economic analysis show that sellers prefer to join the clusters of shops in a costly search market. For instance, Non (2010) analyses a homogeneous product market with one shop cluster and

[13] The Office of Fair Trading (the OFT) issued a detailed report of the study on consumer attitude towards advertising in 2010. The results of the study show *'just under half the consumers would like to see "standardised information" or suppliers using the same terms in advertisements'*. (Office of Fair Trading (2010)) (Annexe H). Moreover, *'two-thirds of people ended up buying the product that was advertised'*, given that firms charged their customers unadvertised fees on the top of the advertised prices. (Office of Fair Trading (2010))

many isolated shops. She finds that in equilibrium clustered shops earn more than stand-alone sellers if the search costs are sufficiently high. Wolinsky (1983) has looked at the location choice of firms which are distributed around a circle in a sequential consumer search market. He has proved that an equilibrium where all firms settle in one place may arise. There is no particular geographical distribution of shops in the sequential search model of Fischer and Harrington Jr (1996). The search costs per cluster vary across consumers, and all consumers experience the same fixed search cost per stand-alone seller. Fischer and Harrington Jr (1996) find that when the degree of product heterogeneity is sufficiently large, clustered firms earn more than their competitors, and stand-alone sellers prefer to join the cluster. Stahl (1982) has analysed the incentives for firms to cluster in a costly search market where firms are evenly spread on a line. He shows that '*the only sufficient condition for a spatial concentration of firms never to take place is that...all commodities are considered perfect substitutes by all consumers*'.

1.2.2 Empirical models

There have not been many attempts to estimate consumer search cost models in the applied economics literature. This is an important omission because costly consumer search implies that the consideration sets vary among consumers. As a result, estimating demand parameters using prices that are constrained by consumer search may lead to significant biases. Therefore, incorporating information about consumer's consideration set is a crucial step in the estimation of structural market models (see Draganska and Klapper, 2010; De los Santos, Hortaçsu, and Wildenbeest, 2011). In some applications, however, it is quite difficult to observe the consideration set of consumers and there the use of theory of optimal consumer search lends itself as a solution to map from observables to consumer's consideration sets (see Hong and Shum, 2006; Moraga-González, Sándor, and Wildenbeest, 2011).

We classify the empirical models on consumer search according to two dimensions: (1) whether products are homogeneous or differentiated and (2) whether consumers search sequentially or non-sequentially. The degree of product heterogeneity affects the data that is necessary to identify consumer search costs. Meanwhile, the search protocol has an impact on model specification.

We start with markets for homogeneous products. A consumer looks for the cheapest product in a homogeneous product market. Firms chose their prices from an equilibrium price distribution. Then a firm sells to a consumer only if the buyer does not sample other firms with lower prices. There is a negative relationship between the search costs and the size of the consideration set of a non-sequentially searching consumer. Hence, a consumer with high search costs buys from a high price charging firm more often than a consumer with low search costs. This is because, the probability that a high search cost consumer finds a cheaper offer is less than the probability that a low search cost consumer finds a lower price. As a result, Hong and Shum (2006) have shown that the cut-off values of the search cost distribution can be identified using just price data. However, Moraga-González, Sándor, and Wildenbeest (2010) warn that the set of the calculated search cost distribution cut-off values does not converge to a continuous search cost distribution if the number of sellers goes to infinity, provided that the price distribution of only one market is observed. This happens because the incremental benefit of a consumer decreases with every additional observation in her consideration set.

When products are differentiated then there are factors other than the price that affect the probability that a firm gets into a consumer's consideration set. This implies that having data on prices only is not enough to identify the search costs of consumers. If a firm charges a high price then the majority of its customers are not necessarily the customers with relatively high search costs. This is because other product characteristics may overcome the negative price effect on the utility. Hence, it is impossible to claim that a specific subset of firms has not been sampled if a consumer buys from a particular seller. The number of possible consumer's consideration sets increases rapidly with the number of alternatives and this complicates the estimation of the model. Honka (2010) has estimated the USA auto insurance demand function using additional information. She has overcome the many-consideration-sets problem by observing the consideration sets and choices of the customers. Meanwhile, Moraga-González et al. (2011) have proposed a method to estimate the search costs by using market share data. They have solved the issue of many consideration sets by employing importance sampling, which makes the numerical optimization feasible.

A sequentially searching consumer starts searching from the alternatives that have the highest expected utility and searches further as long as the additional gain from search is higher than the additional search costs. The expression that equates the gains from an additional search and the search costs, defines the utility cut-off values (*reservation utilities*). The ranking of the reservation utilities is equivalent to the ranking of the expected utilities. Hence, the reservation utilities are important in defining the search order of a consumer and the identification of the search costs. If products are homogeneous and the prices of firms are drawn from the same equilibrium price distribution then consumers sample firms randomly. Thus, the estimation of the search cost distribution with sequentially searching consumers is similar (but not the same) to the estimation of the search cost distribution in a non-sequential search market. The study of Hong and Shum (2006) suggests that the data on prices is sufficient to identify the parameters of the search cost distribution in a homogeneous product market because the probability for a consumer to find the cheapest offer decreases with the search costs.

Product heterogeneity again makes the inference about the search order more difficult. If products only differ in quality, then consumers start searching from the highest quality products and end up with the lowest quality items. This assumption has been used by Hortaçsu and Syverson (2004), who have estimated the search cost distribution for S&P 500 index funds.

If products are horizontally differentiated then there is no single alternative characteristic that can be used to determine the search order. Hence, some additional data are needed to decrease the myriad of search orders. Koulayev (2010) had information on searching and clicking activity of consumers, when he estimated online demand for hotels. Similarly, De los Santos et al. (2011) had information on browsing history and transactions when they studied the search behaviour of consumers. Brynjolfsson, Dick, and Smith (2010) estimated demand for books by observing the sequence in which the offers were presented to a consumer. Meanwhile, Kim et al. (2010) used a specific Amazon ranking index to infer the search order in the estimation of online demand model for camcorders. Dubois and Perrone (2010) used the distance measure and other vertical characteristics of the supermarkets to infer the order in which a household visited the shops.

1.3 Contribution of this thesis

1.3.1 Mergers, cartels and search costs

Competition between sellers is beneficial for a consumer because competition leads to relatively '*low prices, high quality products, a wide selection of goods and services, and innovation*'.[14] A horizontal merger often decreases the intensity of competition. Hence, mergers are considered to be detrimental for consumer welfare and, correspondingly, are carefully examined by competition authorities.

[14] European Commission (2010b)

The total effect of a merger on consumer surplus and the profits of firms depends on the assumptions of the model. For instance, if the costs of the merged firms decrease, the sellers introduce new varieties or new firms enter the post merger market easily, then consumers may be better off after the merger. Furthermore, the profitability of a merger and incentives to merge depend on the degree of product heterogeneity. In two subsequent chapters of this thesis we analyze the effect of costly consumer search on the incentives to merge in a horizontally differentiated product market. The analysis shows that the search order changes after a merger, and it affects the profits of firms and the total welfare.

Armstrong, Vickers, and Zhou (2009) and Zhou (2011) have proved that the search order has a strong effect on equilibrium prices of firms in a differentiated product market. Their analysis shows that if consumers follow a specific predetermined search order then firms charge different prices even if they have the same production costs and their varieties are identical ex ante. The demand elasticity of a seller decreases with its position on the search list. Then the firms that are closer to the end of the search list charge higher prices in equilibrium than their competitors that are closer to the top of the list. Consequently, later visited sellers sell less and often earn less profit than earlier visited firms.

We analyze how merging affects the order in which firms are visited and the incentives to merge in Chapter 2. Merging involves only joint price setting among merged firms, and neither the number of shops nor the number of varieties per shop changes after a merger. Due to the internalization of pricing externalities the merger and the non-merged firms are expected to charge different prices. If the expectations of consumers about prices are different before and after the merger then the search orders also differ. Then both the internalization of pricing externalities and the search order affect equilibrium prices. We show that there is an equilibrium in the post-merger market where the merged firms charge higher prices than their competitors, and consumers visit the merging shops only after all the non-merged sellers are visited.

The post-merger equilibrium quantities are affected by the search order and the price level. Firstly, the price of the merger is higher than the price of its competitors. Therefore, there is a negative price effect on the quantity of every merging firm. Furthermore, positive search costs imply that less consumers reach the merging shops in general if the sellers are at the end of the search order. Thus, the quantity of the merging shops drops down sharply in the post-merger market. If the search costs are sufficiently high then the quantity of the merger drops down a lot and a merger paradox arises: the merging firms earn less than before the merger. Contrarily, the non-merged firms sell more than in the pre-merger market. Thus, the profits of the non-merged firms are higher than the profits of the merging firms and exceed the pre-merger profits.

The post-merger equilibrium in our model is similar to the post-merger equilibrium in the model of Deneckere and Davidson (1985). The post-merger equilibrium prices are higher than the pre-merger equilibrium price and non-merged firms take a free ride in both models. However, the results of our model suggest that a merger may be not profitable if consumer search is costly. It goes without saying that the outcome of our model converges to the outcome of Deneckere and Davidson (1985) if the search cost approaches zero.

Merging firms prefer to overcome the negative effect of the search order on their profits. However, the internalization of pricing externalities makes it almost impossible for a merger to set its price below the price of its non-merged competitors.[15] Then the merger may use other means to raise the attractiveness of its shops and get on the top of the consumers' search list. This issue is analysed in Chapter 3 of the thesis. We assume that a merger starts selling all the varieties of the parent firms in all its shops (or what is equivalent in our model, closes all its shops but one and puts all its varieties in one place). In this way, a consumer can observe many varieties in the shop of the merger for the same search cost that she incurs if she visits a typical non-merged firm. We show that this alters fundamentally the nature of equilibrium since it turns out that in equilibrium consumers start searching at the merging firms if the search cost is sufficiently high.

In the second model the order effect works in favour of the merger, and its profit per variety is higher than in a pre-merger market. However, the profit of a non-merged firm is less than in a pre-merger market if the search cost is sufficiently high. The profit of the merger increases enough to compensate the decrease in the profits of its non-merged counterparts. Thus, the total industry profit goes up after the horizontal integration for any value of the search cost. If the search cost is sufficiently high then a merger raises consumer surplus. This happens because a consumer observes several varieties in the shop of the merger and saves on her total search costs. The savings on search costs outweigh the negative effect of the higher post-merger prices and consumer surplus increases.

Real market examples show that firms may pool their products together or keep

[15] We show in Chapter 2 that there may be an equilibrium when the merger is visited before the non-merged firms because the later charge a higher price than the merger. However, this equilibrium exists just for some parameter values and can be destabilized by introducing some zero search cost consumers.

them separately after a merger. For instance, the EC cleared the merger between *Adidas-Salomon AG* and *Reebok International Ltd* at the beginning of 2006. However, both brands keep their online stores separately and it is not possible to buy *Adidas* sport shoes on the web-site of *Reebok*. On the other hand, a consumer may buy a ticket for the flight that is operated by *AirFrance* on the web-site of *KLM*, or a ticket to the flight that is operated by *Swiss European Air Lines For Swiss* on the web page *Lufthansa*. The choices of merged firms to pool their differentiated products together or keep selling separately indicate whether the search order effect is strong. The clients of the sport-clothing companies are relatively loyal to their favourite brand names. Therefore, an increase in prices probably did not deter consumers from starting their search from the shops of Reebok and Adidas. Moreover, separate selling places make more distinction between brands and firms can charge higher prices. Meanwhile, it may be that a customer cares about available flight times and accessible airports more than the brand names of the airlines. Therefore, all flights of the merged airlines can be found on the same web page.

The incentives to collude and cartel stability in a sequential consumer search market with differentiated products are analysed in Chapter 4. It has been shown by Schultz (2005) that market transparency from the consumer point of view makes cartels less stable. He has analysed a Hotelling model where a fraction of consumers are not informed about prices in the market, whereas the rest of the buyers are perfectly informed. He has shown that the critical discount factor above which a cartel becomes sustainable increases if the fraction of uninformed consumers decreases.

An increase in the costs of search works in the same way as an increase in the fraction of uninformed consumers. This happens because a consumer searches less on average if the search cost goes up. Hence the results of our analysis are in line with the findings of Schultz (2005): a cartel becomes more stable if the search cost goes up. The result is driven by the fact that the deviation price is observed by less consumers if the search cost increases. Then a deviant does not attract as many customers as in a fully transparent market, and the deviation is less attractive.

1.3.2 An empirical model of sequential search for differentiated products

In Chapter 5 of this thesis we develop an empirical model of sequential search for differentiated products. The model borrows heavily from the theory used in earlier chapters. To make the model more appealing in empirical applications, we generalize the standard setup of Wolinsky (1986). In particular we develop an application of such a model for a retail banking industry. A bank is a universal financial institution that is involved in many financial activities. It accepts deposits, issues loans, mortgages, acts as an intermediary in the transactions among individuals, companies and governments, participates in financial markets as an investor, etc. Furthermore, the banking market is quite concentrated, which implies that the unilateral actions of a single bank have a big impact on the market equilibrium. Additionally, the markets for different banking services are closely related to each other, e.g. a bank may accept the deposits of its customers in very unprofitable terms if it makes the funding access to other credit institutions more costly, and allows the credit institution to gain a dominant position in a credit market. Hence, it is a big challenge to derive a full structural banking service market model.

We employ the MNL framework to derive a demand function for saving deposits. We assume that bank customers observe only several saving contract characteristics at zero search costs and the rest of the characteristics have to be learnt at positive search costs. The customers search banks sequentially with perfect recall. It is assumed that bank customers treat the service of banks as a differentiated product. Hence, the estimation requires data on interest rates, the market shares of saving contracts and the characteristics of the banks.[16]

The idea to use discrete choice models for the estimation of the demand of banking services is not a unique idea. Ishii (2005) has used this method to estimate how the demand of saving deposits and bank's incentives to invest in their ATM networks are affected by the compatibility of ATM networks. This method has been applied by Dick (2008) for the deposit demand estimation in the banking industry in the USA. Molnár, Nagy, and Horváth (2007), Nakane, Alencar, and Kanczuk (2006) and Zhou (2008) used the random coefficient MNL demand model to estimate the structural banking market models in Hungary, Brazil and China respectively. Our novelty is that we introduce the aspect of consumer search in this type of models. If the search is costly then a bank customer often accepts alternatives other than the one that gives maximum utility. Therefore, the estimated parameters are biased if positive consumer search costs are ignored in the derivation of the demand function. If the estimated parameter next to the interest rate is less than the true value then the elasticity of the saving deposit demand function is underestimated and the results of a counterfactual merger simulation can be quite

[16] The quantities of accepted saving deposits, their interest rates and other data are reported by credit institutions to the central bank of a particular country. Therefore, this data is actually available, although strict data confidentiality has restrict us so far the access to the necessary information.

misleading.

The search order that consumers follow when they search for banking services has to be known for the correct specification of the demand function. This problem is very severe in horizontally differentiated product markets, because it is impossible to infer the search order from the observable product characteristics. We overcome this problem by assuming that the search cost parameter varies across consumers and banks independently. Thus, any search order may be followed with a positive probability. The number of possible search orders increases with the number of banks.[17] Hence, in applications to markets with many firms the estimation of the model becomes very hard. In those cases, some additional information about consumer search behaviour is needed. In very concentrated markets such as the retail banking, the model is tractable.

At the stage of closing this thesis, we lacked data from a real world bank services market. Because of this, the demand model is estimated using simulated market data. The simulation of the market shares involves the integration over the search cost distribution and solving a large number of non-linear search rule equations. Thus, the simulation of the data and the estimation of the model are very time consuming. In order to perform a sufficient number of estimations we impose a couple of restrictions on consumer search. We assume that some bank customers have zero search costs and the rest of the customers search at most one credit institution if they search at all. The second assumption decreases the number of non-linear equations that have to be solved during the simulation of the data and the estimation of the model. Thus, the simulation and the estimation time significantly decreases.

[17] If there are n banks then a consumer may follow $n!$ search orders.

Chapter 2

Consumer Search Costs and the Incentives to Merge with Bertrand Competition*

2.1 Introduction

One of the most important insights in merger analysis is that merging is not very attractive in the environments where firms compete in quantities and offer similar products (Salant et al. (1983)). This result, known as the *merger paradox*, arises because the output reduction of the merging firms, which favors the coalition partners, is accompanied by an output expansion of the non-merging ones, which hurts them and has a dominating influence. Deneckere and Davidson (1985) show that price-setting firms selling horizontally differentiated products, other things equal, always have an incentive to merge. This result arises because the price increase of the merging firms, which favors the coalition partners, is accompanied by the price increase of the non-merging firms, which also favors them.

While no one would deny that searching for a price and product fit is costly in real-world markets – think for example about the time we spend test-driving new cars, acquiring new furniture, trying on new clothes, etc. – there has been little work in the industrial organization literature on the influence of search costs on the incentives to merge and on the aggregate implications of mergers. In this chapter

*This chapter is based on Moraga-González and Petrikaitė (2011a) we argue that when search costs are important mergers may become unprofitable, even if they involve firms competing in prices and selling horizontally differentiated products.

Our model has a finite number of firms selling differentiated products. The exact utility a buyer derives from consuming a product can only be ascertained upon visiting the seller. Consumers search for satisfactory deals sequentially. Firms compete in prices. In a pre-merger market, all firms look alike and when consumers pick a first shop to visit, they do so in a random way. Those consumers who fail to find a satisfactory product continue searching and once again they pick the next shop to visit randomly; and so on. This model was introduced by Wolinsky (1986) and was further studied by Anderson and Renault (1999). When search cost is equal to zero, the model is similar to Perloff and Salop (1985) and merger analysis gives the same results as in Deneckere and Davidson (1985).

When search cost is positive, higher prices charged by the merging stores result in consumers searching first at the non-merging firms' and then, in the event they fail to find a satisfactory product in those firms, continue searching at the merging stores. In equilibrium, as search costs increase, the share of consumers who walk away from the non-merging stores and show up at the merged shops falls, which makes merging less profitable. We show that any two-firm merger is unprofitable if search costs are sufficiently high. Moreover, we show that any arbitrary k-firm merger becomes unprofitable if search costs and the number of non-merging firms are sufficiently high. These results establish a new merger paradox. What is interesting about this paradox is that it arises even if firms sell horizontally differentiated products and compete in prices.

Janssen and Moraga-González (2007) also study mergers in a consumer search market. In contrast with the present model, they focus on markets where firms sell homogeneous products and there is price dispersion. Their main result is that mergers have redistributive effects with consumers searching little getting better off at the expense of consumers who search a lot. Our model is also related to a recent literature on ordered search. Arbatskaya (2007) studies a market for homogeneous products where the order in which firms are visited is exogenously given. In equilibrium prices must fall as the consumer walks away from the firms visited first. Zhou (2011) considers the case of differentiated products and finds the opposite result. Armstrong et al. (2009) study the implications of "prominence" in search markets. In their model, there is a firm that is always visited first and this firm charges lower prices and derives greater profits than the rest of the firms, which are visited randomly after consumers have visited the prominent firm. Zhou (2009) extends the ideas in Armstrong et al. (2009) to the case in which a set of firms, rather than just one, is prominent. In our model, the merging stores, by raising their prices to internalize the pricing externalities they exert on one another, confer the non-merging firms a "prominent" position in the marketplace. In Haan and Moraga-González (2011), firms gain prominence by advertising. Also related is the paper of Hortaçsu and Syverson (2004), who present a model where sampling probability variation across firms is used to explain price dispersion in the mutual funds industry.

The remainder of the chapter is organized as follows. Section 2.2 describes the consumer search model. Section 2.3 presents the equilibrium analysis, for both cases, the pre-merger market and the post-merger market. Some model modifications and the discussion about other equilibria are in section 2.4. Section 2.5 offers some concluding remarks. Various proofs are placed in an appendix to ease the reading of the chapter.

2.2 The model and the pre-merger symmetric equilibrium

We use Wolinsky (1986) model of search for differentiated products. On the supply side of the market there are $n \geq 3$ firms selling horizontally differentiated products. All firms employ the same constant returns to scale technology of production and we normalize unit production costs to zero. On the demand side of the market, there is a unit mass of consumers. A consumer m has tastes described by an indirect utility function

$$u_{mi}(p_i) = \varepsilon_{mi} - p_i,$$

if she buys product i at price p_i. The parameter ε_{mi} can be thought of as a match value between consumer m and product i. Match values are independently distributed across consumers and products. We assume that the value ε_{mi} is the realization of a random variable uniformly distributed on $[0, 1]$. No firm can observe ε_{mi} so practising price discrimination is not feasible. In what follows we will define $z_\ell \equiv \max\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_\ell\}$. For later reference, it will also be useful to calculate the optimal price of a multi-product monopolist selling ℓ varieties, which we refer to as p_ℓ^m. This price maximizes $p(\Pr[z_\ell \geq p])$, and this gives $p_\ell^m = (1+\ell)^{-\frac{1}{\ell}}$. Setting $\ell = 1$, we have the single-product monopolist, whose price will simply be labeled

p^m.

Consumers search sequentially with costless recall. We assume that a search cost s is relatively small so that the first search is always worth, that is:

$$0 \leq s \leq \bar{s} \equiv \Pr[\varepsilon \geq p^m] E[\varepsilon - p^m \mid \varepsilon \geq p^m],$$

which yields $\bar{s} \equiv (1 - p^m)^2/2$. When the search cost is equal to zero, the model is similar to Perloff and Salop (1985).

We focus on symmetric equilibrium.[1] Following Wolinsky (1986), let p^* denote the price charged by firms other than firm i and consider the (expected) payoff to a firm i which deviates from the symmetric equilibrium by charging a price p_i. Assume $p_i \geq p^*$ without loss of generality.

We start by computing the probability that a consumer accepts the offer of firm i, conditional on visiting firm i first. Suppose that the purchase option at firm i gives the buyer utility $\varepsilon_i - p_i$. If $\varepsilon_i - p_i < 0$, the consumer will search again given our assumption $s < \bar{s}$. Suppose $\varepsilon_i - p_i \geq 0$. In equilibrium, a buyer who contemplates searching again expects to see a price of p^* at the next shop to visit. Therefore, searching one more time, say at firm j, yields gains only if $\varepsilon_j > \varepsilon_i - p_i + p^* \equiv x$, i.e., if the consumer prefers the new option over option i. The expected benefit from searching once more is then

$$\int_x^1 (\varepsilon - x) d\varepsilon = \frac{1}{2}(1 - x)^2 \tag{2.1}$$

Searching one more time is worthwhile if and only if these incremental benefits exceed the cost of search s. The buyer is exactly indifferent between searching once more and stopping and accepting the offer at hand if $x = \overline{x}$, with $\overline{x}$ given by the solution to $\frac{1}{2}(1-x)^2 = s$, i.e., $\overline{x} = 1 - \sqrt{2s}$. Since $s \in \left[0, (1-p^m)^2/2\right]$, we have that $\overline{x} \in [p^m, 1]$.

In any equilibrium $\overline{x} \geq p^*$. Therefore, the probability that a buyer stops search-

[1] We note that asymmetric equilibria can be sustained in this model. The idea is that if consumers believe that firms' prices are, say, ordered as follows $p_1 < p_2 < ... < p_n$, then it is optimal for consumers to visit firms in that order and for firms to price in a way to make consumer beliefs coherent. The unattractive feature of these equilibria is that they are not determined by the underlying characteristics of the market, but by an indeterminacy of beliefs. We will ignore this types of equilibria in our model. A completely different situation is that studied in Zhou (2011) where it is assumed that the shops of the firms are arranged in a particular way so consumers have no alternative than to visit them in a pre-specified exogenous order.

ing at firm i, given that firm i is visited first, is equal to

$$\Pr[x > \bar{x}] = 1 - \bar{x} - p_i + p^*,$$

provided the deviating price is not too high, i.e., $p_i < 1 - \bar{x} + p^*$ for otherwise every single consumer would walk away from firm i.[2]

The consumer may visit firm i after having visited other firm(s). The probability that a consumer goes to firm i in her second search and decides to acquire the offering of firm i right away is $\bar{x}(1 - \bar{x} - p_i + p^*)$.[3] Similarly, the probability that a consumer goes to firm i in her ℓ-th search and decides to acquire the offering of firm i right away is $\bar{x}^{\ell-1}(1 - \bar{x} - p_i + p^*)$.

To complete firm i's payoff calculation, we need to compute the joint probability that a consumer walks away from every single firm in the market and happens to return to firm i to conduct a transaction, that is

$$\Pr[\max\{0, z_{n-1} - p^*\} < \varepsilon_i - p_i < \bar{x} - p^*]$$

This probability is independent of the order in which firms are visited. We will label it as r_a to indicate that these are consumers who return to a firm i after having visited all the firms in the market. We then have:

$$r_a \equiv \int_{p_i}^{\bar{x}+p_i-p^*} (\varepsilon_i - p_i + p^*)^{n-1} d\varepsilon_i = \int_0^{\bar{x}-p^*} (\varepsilon_i + p^*)^{n-1} d\varepsilon_i = \frac{1}{n}(\bar{x}^n - p^{*n}) \quad (2.2)$$

Using the notation introduced above, we can now write firm i's expected profits:

$$\pi_i = p_i \left[\frac{1 - \bar{x}^n}{n(1 - \bar{x})}(1 - \bar{x} - p_i + p^*) + r_a \right]. \quad (2.3)$$

We look for a symmetric Nash equilibrium in prices. After the requirement that consumer expectations are fulfilled, i.e., $p_i = p^*$, the first-order condition (FOC) is:

$$1 - p^{*n} - p^* \frac{1 - \bar{x}^n}{1 - \bar{x}} = 0 \quad (2.4)$$

It is easy to check that (2.4) has a unique solution that satisfies $\bar{x} \geq p^* \geq 1 - \bar{x}$.[4]

[2] In what follows we derive the payoff of a firm under the assumption that $p_i < 1 - \bar{x} + p^*$. When this does not hold, the payoff is slightly different. We deal with this case later (see footnote 4).

[3] Letting j denote the firm visited earlier, this probability is given by $\Pr[\varepsilon_i - p_i > \bar{x} - p^* > \varepsilon_j - p^*]$.

[4] The equilibrium price p^* is indeed an equilibrium if no firm has an incentive to deviate from it. So far we have checked that "small" deviations are not profitable. Suppose now that the deviant firm charges a price so high that consumers always walk away from it and, therefore, this firm only sells

In addition, one can show that the equilibrium price increases in the search cost s (Wolinsky, 1986; Anderson and Renault, 1999).

The profits of a typical firm in the pre-merger situation are

$$\pi^* = \frac{1}{n} p^* (1 - p^{*n}) \tag{2.5}$$

2.3 Equilibrium when k firms merge

In this section we study the price implications of mergers and the incentives to merge. As in Deneckere and Davidson (1985), we abstract from efficiency gains and focus on the effects of joint (price) decision-making. Consider that k firms merge, with $2 \leq k \leq n-1$. In what follows, a typical merging firm will be labeled i, while a typical non-merging store will be labeled j.

As before we focus on symmetric equilibria in the sense that all non-merging firms will be assumed to charge $\tilde{p}^*$, and all merging firms will be supposed to charge $\hat{p}^*$. As it is expected, suppose also that the merging firms charge higher prices than the non-merging firms, i.e., $\tilde{p}^* < \hat{p}^*$. This is reasonable because internalizing the pricing externalities the merging firms confer on one another lead these firms to charge higher prices than the non-merging ones.[5]

Given this, optimal consumer search (see e.g. Kohn and Shavell, 1974) implies consumers should start searching for a satisfactory product at the non-merging

to those consumers who come back to it after having visited all other firms. In that case the deviant profits become $\Pi_i(p_i; p^*) = p_i \int_{p_i}^{1} (\varepsilon_i - p_i + p^*)^{n-1} d\varepsilon_i$. Because of log-concavity of the uniform density function, this profits expression is quasi-concave in own price (Caplin and Nalebuff (1991)). Taking the derivative of the deviating profits with respect to p_i, and setting $p_i = p^*$, we get $d\Pi_i/dp_i|_{p_i=p^*} = (1 - p^{*n} - np^*)/n < 0$, where the inequality follows from the fact that p^* solves (2.4). Since deviating profits are quasi-concave and they decrease at $p_i = p^*$, we conclude they are even lower at prices p_i such that $\overline{x} + p_i - p^* > 1$.

[5] A comment on the existence of other equilibria is in order now. As in the pre-merger market, it may be possible to sustain *asymmetric* equilibria in the sense that distinct non-merging and/or distinct merging firms charge different prices. Again, these asymmetries are not based on any underlying characteristic of the market and can only be sustained because of the indeterminacy of consumer beliefs discussed in the previous section. We will abstract from these types of asymmetric equilibria. What may also happen is that a *symmetric* equilibrium where the merging firms charge a price *lower* than the non-merging firms, i.e. $\tilde{p}^* > \hat{p}^*$, exists. This equilibrium is counterintuitive because we know that joint-profit maximization leads the merging firms to internalize the pricing externalities they impose on one another, which calls for higher rather than lower prices than the non-merging firms. Therefore, if this type of symmetric equilibrium exists, it must be because the force of consumer beliefs more than offsets the effect of joint profit-maximization. Later in Section 2.4.2 we prove that an equilibrium with $\tilde{p}^* > \hat{p}^*$ fails to exist when for example the search cost is low or when the search cost is high. In those cases, the strength of the order effect driven by consumer beliefs is relatively weak. The implication of this result is that if one takes seriously such an equilibrium where the merging firms charge lower prices than the non-merging ones, consumer beliefs should be discontinuous in search costs, which is difficult to justify. In Section 2.4.2, we also show that this type of equilibrium can easily be destabilized when for example there exist consumers in the market who have zero search costs.

firms and then, if no alternative is found to be good enough in those firms, continue searching at the merging ones. To calculate the equilibrium, we proceed by computing the payoff the two types of stores (merging and non-merging) would obtain when deviating from the equilibrium prices. Then we derive the FOCs, require consumer expectations to be correct, and solve for equilibrium prices.

Payoff to a deviant non-merging store.

We now compute the payoff of a non-merging store j that deviates from $\tilde{p}^*$ by charging $\tilde{p} \neq \tilde{p}^*$. As all non-merging firms are supposed to charge the same price $\tilde{p}^*$, consumers are assumed to visit them randomly. Therefore the deviant firm may be visited in first place, second place and so on till the $(n-k)$-th place. As any other non-merging store, the deviant has a probability $1/(n-k)$ of being visited in each of these positions. When the consumer visits the deviant in the 1st, 2nd, ..., $(n-k-1)$-th place, the decision whether to continue searching or not takes into account that the next visited shop is also a non-merging store. By contrast, when the deviant firm is the last non-merging store visited by the consumer, i.e. the $(n-k)$-th, the decision of the consumer is slightly different because the next shop to be visited is a merging store and such a store charges a price different from the price of a non-merging store. Since the consumer stopping rule is different at any of the first $n-k-1$ non-merging stores than that at the last non-merging store, it is convenient to distinguish among those two cases.

- Consider the deviant non-merging firm j that is visited by a consumer in h-th place, with $h = 1, 2, ..., n-k-1$. Suppose the deal a consumer observes upon entering the deviant's shop is $\varepsilon_j - \tilde{p}$. There are three circumstances in which the deviant sells to this consumer.
  - First, the consumer may stop searching at this shop and buy there right away. This occurs when $\varepsilon_j \geq \overline{x} - \tilde{p}^* + \tilde{p}$, where $\overline{x}$ was defined in Section 2.2. Therefore, the joint probability a consumer visits the deviant in h-th place and buys there directly is

$$\Pr[z_{h-1} - \tilde{p}^* < \overline{x} - \tilde{p}^* < \varepsilon_j - \tilde{p}] = \overline{x}^{h-1} (1 - \overline{x} + \tilde{p}^* - \tilde{p})$$

  - Second, the consumer may walk away from the firm visited in h-th place and come back to it after visiting all non-merging stores. To see this, note that optimal search implies that the consumer would walk away from the last non-merging store to visit one of the merging firms if $z_{n-k} \leq \bar{x} - \hat{p}^* + \tilde{p}^*$. Moreover, if the consumer did arrive to the $(n-k)$-th non-merging store it is because $z_{n-k-1} \leq \bar{x}$. Since $\hat{p}^* > \tilde{p}^*$, it is clear that the condition to leave the last non-merging store and continue searching among the merging stores is more stringent than to continue searching among the non-merging stores. For this reason, the consumer may return to the deviant firm after having visited all non-merging firms and buy there. This occurs when

$$\Pr[\max\{z_{n-k-1} - \tilde{p}^*, \overline{x} - \hat{p}^*\} < \tilde{\varepsilon}_j - \tilde{p} < \overline{x} - \tilde{p}^*]$$

and this gives the following "coming back" or "returning" demand:

$$\tilde{r}_{nm} \equiv \int_{\overline{x}-\hat{p}^*+\tilde{p}}^{\overline{x}-\tilde{p}^*+\tilde{p}} \left(\varepsilon_j - \tilde{p} + \tilde{p}^*\right)^{n-k-1} d\varepsilon_j = \frac{1}{n-k}\left(\overline{x}^{n-k} - \left(\overline{x} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\right)$$

where the subindex "*nm*" refers to the fact that consumers return to the deviant firm after having visited all the non-merging stores.

– Finally, the consumer may walk away from the deviant non-merging firm and come back to it after having visited all the firms in the market. This occurs when

$$\Pr[\max\{z_{n-k-1} - \tilde{p}^*, z_k - \hat{p}^*, 0\} < \tilde{\varepsilon}_j - \tilde{p} < \overline{x} - \hat{p}^*]$$

and this gives the following “coming-back” demand

$$\tilde{r}_a \equiv \int_0^{\overline{x}-\hat{p}^*} \left(\varepsilon_j + \tilde{p}^*\right)^{n-k-1} \left(\varepsilon_j + \hat{p}^*\right)^k d\varepsilon_j \tag{2.6}$$

where the subindex "*a*" again refers to the fact that consumers return after having visited *all* the stores.

- We now consider the case in which the deviant firm is visited in $(n-k)$-th place. This type of firm sells to the consumers in two cases:

  – First, the consumer stops searching at this shop and buys there right away. This occurs with probability

$$\Pr[\varepsilon_j - \tilde{p} \geq \max\{z_{n-k-1} - \tilde{p}^*, \overline{x} - \hat{p}^*\} \text{ and } z_{n-k-1} < \overline{x}]$$

and this gives a demand

$$\overline{x}^{n-k-1}\left(1-\overline{x}+\tilde{p}^{*}-\tilde{p}\right)+\frac{1}{n-k}\left[\overline{x}^{n-k}-\left(\overline{x}-\hat{p}^{*}+\tilde{p}^{*}\right)^{n-k}\right]$$

– Second, the consumer walks away from this firm and comes back to it after visiting all the firms in the market. In this second case we have exactly the same expression for returning consumers as in (2.6).

As a result, taking into account the different positions in which the deviant firm may be visited by a consumer, we get the profits of a deviant non-merging firm:

$$\tilde{\pi}=\tilde{p}\left[\frac{1}{n-k}\frac{1-\overline{x}^{n-k}}{1-\overline{x}}\left(1-\overline{x}+\tilde{p}^{*}-\tilde{p}\right)+\tilde{r}_{nm}+\tilde{r}_{a}\right] \tag{2.7}$$

Payoff to a deviant merging store.

We now compute the joint payoff of the merging stores. Recall that since consumers expect the price set at the merging firms $\hat{p}^*$ to be greater than $\tilde{p}^*$, they postpone visiting them until they have visited all the non-merging firms. Suppose that the merging firms deviate by charging $\hat{p} \neq \hat{p}^*$.

Consider a consumer who walks away form the last non-merging store and observes a deal $\varepsilon_i - \hat{p}$ at the first merging store she visits. We note first that such a consumer will never return to any of the non-merged firms without first visiting all other merging stores. This is because the utilities from all non-merged firms are lower than $\bar{x} - \hat{p}^*$, which is exactly the reservation utility at any of the merging shops. We now ask whether the consumer will continue searching after she visits the first merging shop. Clearly, she will continue searching when her best deal so far does not give her sufficiently high utility. She will do the same when the highest utility so far is obtained at one of the non-merging stores, that is, $z_{n-k} - \tilde{p}^* > \varepsilon_i - \hat{p} > 0$. In case the best deal is the one at the merging store, the consumer will continue searching when $\varepsilon_i - \hat{p} < \bar{x} - \hat{p}^*$.[6] As a result, the probability that the

[6] We assume that a buyer who observes a deviation price $\hat{p}$ at one of the merging stores does not change the expectation that the other merged firms charge $\hat{p}^*$. This assumption is adopted for technical reasons though it can easily be justified because consumers usually need not know both whether the pricing of firms is joint or independent, or the ownership structure of the firms. If consumers did know the merging firm deviates jointly in all its stores, then they would update their expectations correspondingly and this would lead to a kink in the demand function of the merging firms. In this situation, downward deviations lead to exactly the same payoff as here. However, upward deviations lead to a different payoff function, which suggests the existence of multiple equilibria.

consumer arrives at the first merging store and buys there right away is

$$\Pr\left[\varepsilon_i - \hat{p} \geq \overline{x} - \hat{p}^* \geq \max\{z_{n-k} - \tilde{p}^*\right] = \left(\overline{x} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\left(1 - \overline{x} + \hat{p}^* - \hat{p}\right)$$

Suppose now the consumer walks into the h-th merged store, $h = 2, ..., k$. The probability this consumer buys at that shop right away is

$$\Pr[\max\{z_{h-1} - \hat{p}, z_{n-k} - \tilde{p}^*\} < \overline{x} - \hat{p}^* < \varepsilon_i - \hat{p}]$$

This gives a demand $\left(\overline{x} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\left(\bar{x} - \hat{p}^* + \hat{p}\right)^{h-1}\left(1 - \overline{x} + \hat{p}^* - \hat{p}\right)$. Taking into account that k firms belong to a merger, the probability that a consumer terminates her search in one of k merged firms equals

$$\begin{aligned}
&\left(\bar{x} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\sum_{h=1}^{k}\left(\bar{x} - \hat{p}^* + \hat{p}\right)^{h-1}\left(1 - \bar{x} + \hat{p}^* - \hat{p}\right) \\
&= \left(\bar{x} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\left[1 - \left(\bar{x} - \hat{p}^* + \hat{p}\right)^{k}\right]
\end{aligned}$$

Some consumers visit all shops in the market and decide to return to one of the merging stores to conduct a purchase. Then, the joint returning demand obtained by the merging stores is given by:

$$\sum_{h=1}^{k}\Pr\left[\varepsilon_i \geq \max\left\{z_{k-1},\, z_{n-k} + \hat{p} - \tilde{p}^*,\, \hat{p}\right\} \text{ and } \varepsilon_i < \bar{x} - \hat{p}^* + \hat{p}\right]$$

which gives the following "coming-back" demand

$$\hat{r}_a \equiv k\int_0^{\bar{x}-\hat{p}^*}\left(\varepsilon_i + \tilde{p}^*\right)^{n-k}\left(\varepsilon_i + \hat{p}\right)^{k-1}d\varepsilon_i \tag{2.8}$$

The payoff to a deviating merged entity is then:

$$\hat{\pi} = \hat{p}\left[\left(\bar{x} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\left(1 - \left(\bar{x} - \hat{p}^* + \hat{p}\right)^{k}\right) + \hat{r}_a\right] \tag{2.9}$$

2.3.1 An example with three firms

In order to develop some intuition for the main results, we briefly discuss in this subsection an example with three firms in which two of them merge. Prices and profits in the pre-merger market follow straightforwardly from setting $n = 3$ in (2.4) and (2.5).

Firms payoffs in the post-merger market are

$$\tilde{\pi} = \tilde{p}\left[1 - \overline{x} - \tilde{p} + \hat{p}^{*} + \frac{1}{3}(\overline{x}^{3} - \hat{p}^{*3})\right]$$

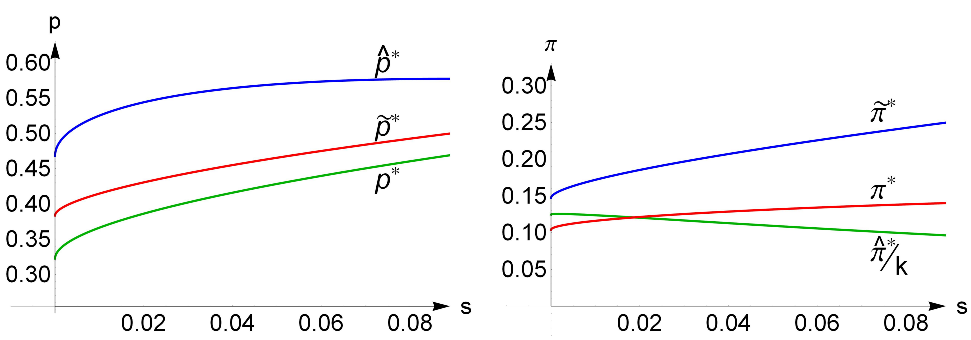
$$\hat{\pi} = \hat{p}\left[(\overline{x} - \hat{p}^{*} + \tilde{p}^{*})\left(1 - (\overline{x} - \hat{p}^{*} + \hat{p})^{2}\right) + 2\int_{\hat{p}}^{\overline{x} - \hat{p}^{*} + \hat{p}} \varepsilon(\varepsilon - \hat{p}^{*} + \tilde{p}^{*})d\varepsilon\right]$$

with corresponding FOCs after applying symmetry

$$1 - 2\tilde{p}^{*} + \hat{p}^{*} - \overline{x} + \frac{1}{3}\left(\overline{x}^{3} - \hat{p}^{*3}\right) = 0 \tag{2.10}$$

$$\frac{4\hat{p}^{*3}}{3} + \tilde{p}^{*} - 3\hat{p}^{*2}\tilde{p}^{*} + \overline{x} - \frac{\overline{x}^{3}}{3} - \hat{p}^{*}\left(1 + \overline{x}^{2}\right) = 0 \tag{2.11}$$

It can be seen that this system of equations has always a solution and that such solution is unique and constitutes a Nash equilibrium.[7] For a fixed search cost, it is straightforward to solve the FOCs numerically for equilibrium prices. The results are given in Figure 2.1a, where we plot post-merger equilibrium prices, $\hat{p}^{*}$ and $\tilde{p}^{*}$, against search costs. For comparison purposes, we also plot the pre-merger price, p^{*}. As expected, all prices are increasing in search costs. As searching for price and product fit becomes more costly, firms have more market power over the consumers who pay them a visit and this results in higher prices for all the firms.



(a) Prices (b) Profits

Figure 2.1. Pre- and post-merger prices, and merger profitability.

As the graph reveals, post-merger prices, whether from merging or non-merging firms, happen to be *higher* than the pre-merger price. This deserves a comment. In our model, the non-merging firm is expected to charge a price lower than that of the merging stores and therefore it is visited first by the consumers; after that, if

[7] Further, we can show that no other symmetric equilibrium exists (see Proposition 2.5(c)).

consumers do not find a satisfactory product there, they proceed by checking the goods on sale at the merging stores. In the terminology of Armstrong et al. (2009), what happens in our model is that by merging, the joining firms end up conferring *market prominence* to the non-merging store.

Armstrong et al. (2009) study the effects of market prominence. They show, on the one hand, that a prominent firm charges a lower price than the rest of the firms, as it is here the case. In their model this price ranking originates from the order in which firms are visited by consumers. The firm that is visited first has a more elastic demand than the other firms just because the latter receive consumers who were dissatisfied at the first firm so in effect it is as if they had fewer acceptable options. In addition, Armstrong et al. (2009). show that the prominent firm charges a lower price than in the case in which no prominent firm exists. This result does not arise here. The reason is that in our model there is a second force counteracting with the search-order effect: merging firms internalize pricing externalities between them and raise prices over and above the prices they would charge if they were operating independent stores. This weakens competition further and then all prices increase over and above the status quo (pre-merger) situation.

Figure 2.1b shows how the profits of a merging firm and a non-merging firm, $\hat{\pi}^*/k$ and $\tilde{\pi}^*$, vary with search costs. In addition, the figure gives the pre-merger profits, π^*, so we can readily assess whether merging is worthwhile for the merging parties. The most important point to make here is that the profits of a merging firm decline as the search cost goes up. The reason is that, as the search cost increases, fewer consumers walk away from the non-merging firm and visit the merging firms. This has a major implication on merger profitability: for sufficiently large search costs, merging is not individually rational for the merging entities. This result resembles the well-known *merger paradox*, but its novelty is that it arises under price competition with differentiated product sellers. The graph reveals that the non-merging firm gets a free ride and this ride is freer the higher the search cost. These insights can be proven more generally and this will be the purpose of the next section and the appendix.

2.3.2 Main results

We first study the existence of equilibrium. For this, we first define a critical search cost value below which all firms, including the merging firms, receive positive demand. Let

$$\tilde{s}_k \equiv \Pr[\varepsilon \geq p_k^m] E[\varepsilon - p_k^m \mid \varepsilon \geq p_k^m].$$

$\tilde{s}_k$ is the search cost that makes a consumer who has currently found no acceptable option indifferent between staying at home and visiting a monopoly firm that controls k stores. Since $p_k^m = (1+k)^{-\frac{1}{k}}$, $\tilde{s}_k = (1-(1+k)^{-\frac{1}{k}})^2/2$. In what follows, if a merger of k firms occurs, we will pay only attention to search costs in the set $[0, \tilde{s}_k]$ (or $\bar{x} \in [p_k^m, 1]$). This ensures that each of the k merging stores has positive demand.

Taking the first order derivatives of the payoffs in (2.7) and (2.9) with respect to deviation prices $\hat{p}$ and $\tilde{p}$ respectively and applying the equilibrium requirement that consumer beliefs are correct, i.e., $\hat{p}^* = \hat{p}$ and $\tilde{p}^* = \tilde{p}$, yields the following FOCs:

$$(\bar{x} - \hat{p}^* + \tilde{p}^*)^{n-k}\left(1 - \bar{x}^k - k\hat{p}^*\bar{x}^{k-1}\right) + k\int_0^{\bar{x}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k}(\varepsilon + \hat{p}^*)^{k-2}(\varepsilon + k\hat{p}^*)\, d\varepsilon = 0 \tag{2.12}$$

$$1 - \frac{1-\bar{x}^{n-k}}{1-\bar{x}}\tilde{p}^* - (\bar{x} - \hat{p}^* + \tilde{p}^*)^{n-k} + (n-k)\int_0^{\bar{x}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1}(\varepsilon + \hat{p}^*)^k\, d\varepsilon = 0 \tag{2.13}$$

Proposition 2.1. *Assume that k firms merge. For any $s \in [0, \tilde{s}_k]$, there exists a Nash equilibrium in the post-merger market where consumers start searching at the non-merging stores and then they proceed by searching at the merged ones. Merging firms charge a price $\hat{p}^*$ and the non-merging stores charge a price $\tilde{p}^*$; these prices are given by the unique solution to the system of FOCs (2.12)-(2.13) and the price ranking is consistent with consumer search behavior, that is, $\hat{p}^* > \tilde{p}^*$.*

The proof of this proposition, which is presented in the appendix, has the following steps. We first show that there is a unique pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies the FOCs. We then show that these prices satisfy the inequality $\hat{p}^* > \tilde{p}^*$, which immediately implies that the hypothesized consumer search behavior is optimal.

We explore next the relationship between the post-merger equilibrium prices and the pre-merger equilibrium price.

Proposition 2.2. *The ranking of pre- and post-merger equilibrium prices is $p^* < \tilde{p}^* < \hat{p}^*$ whenever one of the following conditions holds: (a) the search cost is sufficiently low, (b) the search cost is sufficiently high, (c) the number of firms $n = 3$.*

Whether the post-merger equilibrium prices are higher or lower than the pre-merger equilibrium price is a priori ambiguous. Consider the price charged by the non-merging firms. The fact that the potentially merging stores by actually merging confer a "prominent" position in the marketplace to the non-merging stores causes a direct downward pressure on the price of the non-merging firms. When a firm becomes prominent its pool of consumers becomes more elastic. In addition there are two indirect effects. The first is that, since the merging firms are relegated to the last positions of the queue consumers follow when they search, they tend to raise their prices. This effect arises because for the non-prominent firms it holds the opposite, namely, that their demand becomes less elastic. By strategic complementarity, this weakens competition in the marketplace and the non-merging firms tend to raise their prices as well. The last effect follows from the fact that the merging firms internalize the pricing externalities they confer on one another. This also tends to raise their price, and indirectly, by strategic complementarity again, the prices of the non-merging firms. Similar considerations apply to the price of the merging firms. Our proposition shows that when the search-order effects are not very strong then we are sure that the prices increase after a merger. We note however that solving numerically the model we have found no example in which this does not happen. Basically, what we always observe is similar to what happens in Figure 2.1a when $n = 3$.

We study next the relationship between the post-merger equilibrium prices and search costs. The following result extends those in Wolinsky (1986) and Anderson and Renault (1999) about how search costs influence the symmetric equilibrium price to the merger situation studied here.

Proposition 2.3. *The post-merger equilibrium prices $\hat{p}^*$ and $\tilde{p}^*$ increase in search costs.*

Our final result explores merger profitability.

Proposition 2.4. *Assume that the search cost $s \in [0, \tilde{s}_k]$. Then:*
(A) Any 2-firm merger is not profitable if search cost is sufficiently high.
(B) Any arbitrary k-firm merger is not profitable if search cost and the number of competitors are sufficiently high.
(C) If search costs are sufficiently small, any arbitrary k-firm merger is profitable.

As expected, the case in which the search cost is small reproduces naturally the situation in Deneckere and Davidson (1985). However, as search costs increase, fewer consumers walk away from the non-merging stores and visit the merged ones. This lowers the profits of the merging firms. Eventually, when the search cost becomes relatively high, unless there are many firms in the industry and the merger comprises almost all of them, merging becomes unprofitable. The interest of the last proposition is that it puts forward a new *merger paradox*, which arises under price competition with differentiated products. The underlying reason is based on search costs, something quite different from the merger paradox of Salant et al. (1983), which concerns competition with decision variables that are strategic substitutes.

2.4 Discussion

2.4.1 Alternative distributions of consumer tastes

So far we have assumed that consumer tastes are uniformly distributed on the unit interval. In this section we study the sensitiveness of our results to this assumption. For this purpose, we assume that the distribution of match values is $F(\varepsilon) = \varepsilon^{\lambda}$ with $\varepsilon \in [0,1]$ and $\lambda \geq 1$. The parameter λ is a shifter of the distribution of consumer preferences towards the right end of the interval of possible match values. When λ goes up, products become more homogeneous and, therefore, competition between firms becomes fiercer. Note also that the average willingness to pay, which is $\lambda/(1+\lambda)$, also increases in the parameter λ.

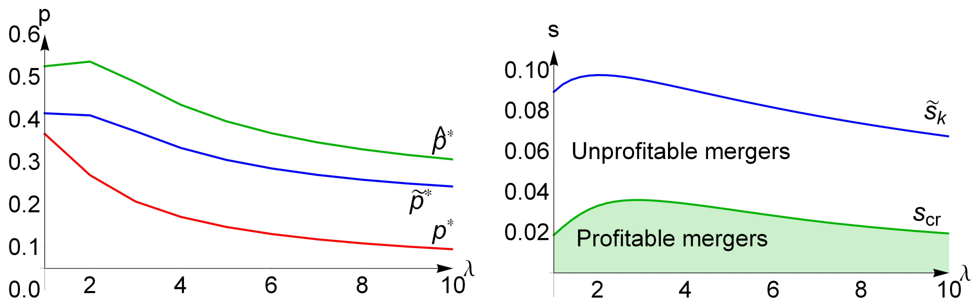
With this distribution of match values, the price of a monopolist selling ℓ varieties becomes $p_{\ell}^{m} = (1+\lambda\ell)^{-1/\lambda\ell}$. The threshold match value $\tilde{x}$ above which consumers stop searching becomes the solution to $\int_{\tilde{x}}^{1} (\varepsilon - \tilde{x})\, d\varepsilon^{\lambda} = s$. This gives a critical search cost $\tilde{s}_{\ell} = \frac{\lambda}{1+\lambda} - p_{\ell}^{m} + \frac{1}{1+\lambda}(p_{\ell}^{m})^{\lambda+1}$ beyond which no consumer would ever visit the shops of a monopoly firm in control of ℓ shops. One can easily check that when $\lambda = 1$ we get the same expressions for the monopoly price and maximum search cost we had in section 2.2.

For simplicity, let us assume that $n = 3$, as in section 2.3. It is easy to rewrite the payoffs of the firms using the new distribution of match values. These are

$$\begin{aligned}
\tilde{\pi}(\tilde{p}, \hat{p}^{*}) &= \tilde{p}\left[1 - (\tilde{x} + \tilde{p} - \hat{p}^{*})^{\lambda} + \int_{\tilde{p}}^{\tilde{x} - \hat{p}^{*} + \tilde{p}} \lambda\varepsilon^{\lambda-1}\left(\varepsilon - \tilde{p} + \hat{p}^{*}\right)^{2\lambda} d\varepsilon\right] \\
\hat{\pi}\left(\tilde{p}^{*}, \hat{p}\right) &= \hat{p}\left[\left(1 - (\tilde{x} - \hat{p}^{*} + \hat{p})^{2\lambda}\right)\left(\tilde{x} + \tilde{p}^{*} - \hat{p}^{*}\right)^{\lambda}\right. \\
&\quad \left. +2\int_{\hat{p}}^{\overline{x} - \hat{p}^{*} + \hat{p}} \lambda\varepsilon^{2\lambda-1}(\varepsilon - \hat{p} + \tilde{p}^{*})^{\lambda} d\varepsilon\right]
\end{aligned}$$

As above, equilibrium prices can be found by taking the FOCs, setting $\hat{p} = \hat{p}^{*}$ and $\tilde{p} = \tilde{p}^{*}$ and solving for $\hat{p}^{*}$ and $\tilde{p}^{*}$. Even in this simple environment with three firms, calculations get complicated when λ can vary freely.

Figure 2.2a shows the pre-merger and post-merger equilibrium prices of Proposition 2.1. Initially there are three firms and then two of them merge. The search cost is fixed at $s = 0.01$ and λ varies from 1 to 10. The graph shows that no matter the value of λ, the price of the non-merging firms is lower than the price of the merging firms. Moreover, we see that as λ goes up, prices increase and then decrease. To understand this, recall that as λ goes up the intensity of competition increases and this tends to lower prices. At the same time, as λ goes up, the average willingness-to-pay also increases and this pushes prices up. The first effect dominates for high levels of λ, while it happens otherwise for low levels of λ.



(a) Prices (b) Merger profitability

Figure 2.2. Pre- and post-merger prices and merger profitability.

In Figure 2.2b we show the region of search costs s and taste parameters λ for which merging is individually rational. The critical search cost level below which merging is profitable, denoted s_{cr}, first increases and then decreases in λ. This result is intimately linked to the price result discussed above. When λ is small, an increase in λ increases the price of the merging stores and this raises merger profitability. Eventually when λ is relatively large, further increases in λ make products too similar and this fosters competition. In this case, as λ increases a smaller search cost is needed to make merging unprofitable.

2.4.2 Other symmetric equilibria

All the discussion so far has been on a symmetric equilibrium with the merging firms charging a higher price than the non-merging firms and, correspondingly, consumers starting their search for satisfactory products at the non-merging stores. This equilibrium is the natural extension of the equilibrium that arises under perfect information (Deneckere and Davidson (1985)) and exists for all (reasonable) levels of the search cost. For this reason, it has been the focal point of the analysis so far.

However, as we mentioned above when we discussed the potential problems associated with the indeterminacy of consumer beliefs about which type of firms charge the lowest prices (cf. footnote 5), another symmetric equilibrium can be proposed. In such alternative symmetric equilibrium, consumers hold the belief that the merging stores charge prices lower than those of the non-merging firms and, correspondingly, they start their search for satisfactory products at the merging firms; firms respond by setting prices in such a way that consumer beliefs are fulfilled.

In this section we focus attention on such an alternative equilibrium. Our first observation is that this alternative symmetric equilibrium is somewhat counterintuitive. The reason is that, due to the internalization-of-pricing externalities effect, we expect the merging firms to charge higher, rather than lower, prices than the non-merging firms. Therefore, if such an equilibrium exists, it must be because the power of consumer beliefs at dictating firm pricing is sufficiently strong so as to more than offset the internalization-of-pricing externalities effect. Can this occur for all parameters? We do not expect it. From the received theory we know that when the search cost is exactly equal to zero (Deneckere and Davidson, 1985), such price ranking is impossible. By "continuity" we expect this alternative equilibrium to fail to exist when the search cost is positive but small. This is indeed what our next result shows. In addition, we can show that the same is true when the search cost is very high or for example when the number of firms is 3.

Proposition 2.5. *Assume that k firms merge. Then a symmetric Nash equilibrium where $\hat{p}^* < \tilde{p}^*$ so that consumers start searching at the merged stores and then proceed by searching at the non-merging stores does not exist if (a) the search cost is sufficiently low, (b) the search cost is sufficiently high and (c) $n = 3$. In these cases, the equilibrium in Proposition 2.1 is unique.*

The proof of this result is in the appendix. There we first derive the payoff functions of the merging and non-merging firms assuming that $\hat{p}^* < \tilde{p}^*$. Then we show that when the search cost is either sufficiently high or sufficiently low, profit maximizing firms charge prices such that $\hat{p}^* > \tilde{p}^*$, which leads to a contradiction. This result implies that the alternative equilibrium where merging firms are visited first can only exist for intermediate levels of the search cost, which casts doubts about the appeal of the equilibrium. In fact, taking such an alternative equilibrium seriously requires consumer beliefs to be discontinuous in search costs, which is difficult to justify.

The intuition behind Proposition 2.5 is as follows. The price ranking of the firms is the outcome of the tension between the search-order effect, which pushes merging firms that are visited first to lower prices relative to the non-merging firms, and the internalization-of-pricing-externalities effect, which works in the opposite direction. The magnitude of the search cost affects the outcome of this tension. In fact, note that the search-order effect is practically non-existent when the search cost is arbitrarily close to zero, while the internalization-of-pricing-externalities effect is the strongest. In this case, the second effect has a dominating influence and this explains the result. When the search cost increases, the search-order effect gains importance, while the internalization-of-pricing-externalities effect loses strength. For intermediate levels of the search cost the equilibrium may exist (though not necessarily as demonstrated for the case $n = 3$). Finally, when search costs are very high, prices, whether form merging or not merging firms, are close to monopoly prices and the search-order effect loses again against the internalization-of-pricing-externalities effect.

We have explored alternative ways to affect the trade-off between the search-order effect and the internalization-of-pricing-externalities effect and rule out the alternative equilibrium where the merging firms charge lower prices and are visited first. What is important is to weaken the power consumer beliefs have at dictating equilibrium prices. For example, one can show that this equilibrium fails to exist when there is a sufficiently large number of consumers who have perfect information. The equilibrium in Proposition 2.1 by contrast survives this modification as well as our main result in Proposition 2.4.

To illustrate this last point, we provide next the outcome of the numerical analysis of a slightly modified model where we partition the set of consumers into two fractions: a fraction of the consumers μ have zero search costs while the rest of the consumers have positive search costs. The rest of the model remains exactly the same. In Figure 2.3 we set $n = 7$ and plot the (blue) region of parameters for which an equilibrium where a two-firm merger charges a price lower than the price of the non-merging firms exists. The graph clearly shows the observation in Proposition 2.5 that, when all consumers have positive search costs, this equilibrium only exists for intermediate levels of the search cost. As we increase the fraction of consumers who have zero search cost, the equilibrium fails to exist. The equilibrium in Proposition 2.1 by contrast exists for all parameters and it can be seen that no matter the level of μ, a merger is unprofitable provided that the search cost is sufficiently large.

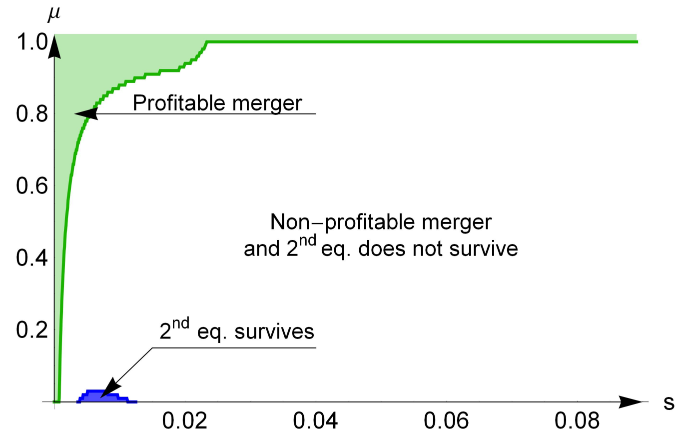


Figure 2.3. Model with μ percent of zero-search-cost consumers

2.5 Concluding remarks

This chapter has studied the role of search costs for merger profitability. We have used a model where firms compete in prices to sell differentiated products and consumers search sequentially to find price and product fit information. When the search cost is set equal to zero, the model gives the results in Deneckere and Davidson (1985). But when search costs are sizable, the price divergence between merging and non-merging firms has implications for the path consumers follow when they search for satisfactory products. Since optimal consumer search prescribes the consumers to visit first the cheaper stores, the merging firms lose a lot of customers when search costs are relatively high. Our main result is that merging becomes unprofitable when the search cost is sufficiently large. The analysis thus shows that a merger paradox can also arise when firms compete in prices to sell differentiated products. The paradox arises because a merger pushes the merging stores all the way back in the search order of consumers.

In the analysis of this chapter we have followed the tradition and studied the implications of joint-decision making. By doing this, the model has focussed on the short-run effects of mergers and has therefore abstracted from a number of issues that are relevant to mergers in the long-run. These issues include all types of business organizational changes aimed at delivering cost reductions. Arguably, in situations where search costs are important, these organizational changes may even include the shutting down of shops and the crowding of products together, which have the potential to generate beneficial search economies for the consumers. These long-run considerations are studied in a subsequent chapter. There we show that search cost economies may render a merger beneficial for consumers.

2.A Appendix

Proof of Proposition 2.1. The proof of the proposition is organized in three claims. The first claim shows that there is a pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies the system of first order conditions (2.12) and (2.13). The second claim shows that such a pair of prices is unique. Finally, the third claim demonstrates that $\tilde{p}^* < \hat{p}^*$.

Claim 2.A.1. *There is at least one pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies (2.12) and (2.13).*

Proof. We first rewrite the FOC (2.12) as $G(\hat{p}, \tilde{p}) = 0$, where

$$G(\hat{p}, \tilde{p}) \equiv \frac{1-\overline{x}^k}{k\overline{x}^{k-1}} - \hat{p} + g(\hat{p}, \tilde{p}) \tag{2.A.1}$$

and

$$g(\hat{p}, \tilde{p}) \equiv \frac{\int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-2} (\varepsilon+\tilde{p})^{n-k} (\varepsilon+k\hat{p})\, d\varepsilon}{(\overline{x}-\hat{p}+\tilde{p})^{n-k}\, \overline{x}^{k-1}}$$

The FOC $G(\hat{p}, \tilde{p}) = 0$ defines an implicit relationship between $\hat{p}$ and $\tilde{p}$. Let the function $\eta_1(\tilde{p})$ define this relationship. This function is represented in Figure 2.A.1 below. By the implicit function theorem we have

$$\frac{\partial \eta_1(\tilde{p})}{\partial \tilde{p}} = -\frac{\partial G/\partial \tilde{p}}{\partial G/\partial \hat{p}} = -\frac{\partial g/\partial \tilde{p}}{\partial g/\partial \hat{p} - 1}, \tag{2.A.2}$$

The numerator of (2.A.2) is positive. This is because

$$\frac{\partial g}{\partial \tilde{p}} = \frac{n-k}{\overline{x}^{k-1}(\overline{x}-\hat{p}+\tilde{p})^{n-k+1}} \int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-2} (\varepsilon+k\hat{p}) (\varepsilon+\tilde{p})^{n-k-1} (\overline{x}-\hat{p}-\varepsilon)\, d\varepsilon > 0$$

The denominator of (2.A.2) is however negative. To see this, we note first that

$$\begin{aligned}
&\bar{x}^{k-1}(\overline{x}-\hat{p}+\tilde{p})^{n-k+1}\left(\frac{\partial g}{\partial \hat{p}} - 1\right) \\
&= (n-k)\int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-2} (\varepsilon+\tilde{p})^{n-k} (\varepsilon+k\hat{p})\, d\varepsilon \\
&+ (\bar{x}-\hat{p}+\tilde{p})(k-1)\int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-3} (\varepsilon+\tilde{p})^{n-k} (2\varepsilon+k\hat{p})\, d\varepsilon \\
&- (\bar{x}-\hat{p}+\tilde{p})^{n-k+1}\, \bar{x}^{k-2}\left[2\bar{x}+(k-1)\hat{p}\right].
\end{aligned} \tag{2.A.3}$$

Assuming $k > 2$, let us take the derivative of the RHS of (2.A.3) with respect to $\bar{x}$. We obtain

$$
\begin{aligned}
&(n-k)\,\bar{x}^{k-2}\,(\bar{x}-\hat{p}+\tilde{p})^{n-k}\,[\bar{x}+(k-1)\,\hat{p}] \\
&+(k-1)\int_0^{\bar{x}-\hat{p}}(\varepsilon+\hat{p})^{k-3}\,(\varepsilon+\tilde{p})^{n-k}\,(2\varepsilon+k\hat{p})\,d\varepsilon \\
&+(k-1)\,(\bar{x}-\hat{p}+\tilde{p})^{n-k+1}\,\bar{x}^{k-3}\,[2\bar{x}+(k-2)\,\hat{p}]-2\,(\bar{x}-\hat{p}+\tilde{p})^{n-k+1}\,\bar{x}^{k-2} \\
&-(n-k+1)\,(\bar{x}-\hat{p}+\tilde{p})^{n-k}\,\bar{x}^{k-2}\,[2\bar{x}+(k-1)\,\hat{p}] \\
&-(\bar{x}-\hat{p}+\tilde{p})^{n-k+1}\,(k-2)\,\bar{x}^{k-3}\,[2\bar{x}+(k-1)\,\hat{p}]
\end{aligned}
$$

which can be simplified to

$$
\begin{aligned}
&-\bar{x}^{k-2}\,(\bar{x}-\hat{p}+\tilde{p})^{n-k}\,[\bar{x}\,(n-k+2)+(k-1)\,\hat{p}] \\
&+(k-1)\int_0^{\bar{x}-\hat{p}}(\varepsilon+\hat{p})^{k-3}\,(\varepsilon+\tilde{p})^{n-k}\,(2\varepsilon+k\hat{p})\,d\varepsilon
\end{aligned}
\tag{2.A.4}
$$

If we now take the derivative of (2.A.4) with respect to $\bar{x}$ we obtain

$$
\begin{aligned}
&-\left[(k-2)\,\bar{x}^{k-3}\,(\bar{x}-\hat{p}+\tilde{p})^{n-k}+(n-k)\,(\bar{x}-\hat{p}+\tilde{p})^{n-k-1}\,\bar{x}^{k-2}\right] \\
&\cdot[\bar{x}\,(n-k+2)+(k-1)\,\hat{p}]-(n-k+2)\,\bar{x}^{k-2}\,(\bar{x}-\hat{p}+\tilde{p})^{n-k} \\
&+(k-1)\,\bar{x}^{k-3}\,(\bar{x}-\hat{p}+\tilde{p})^{n-k}\,[2\bar{x}+(k-2)\,\hat{p}]
\end{aligned}
$$

Putting terms together and simplifying, this equals to

$$-(n-k)\,\bar{x}^{k-2}\,(\bar{x}-\hat{p}+\tilde{p})^{n-k-1}\,[\bar{x}\,(n+1)+\tilde{p}\,(k-1)]<0.$$

This implies that the derivative of the RHS of (2.A.3) with respect to $\bar{x}$, given in equation (2.A.4), is decreasing in $\bar{x}$. Setting $\bar{x}$ equal to its lowest value, $\hat{p}$, in (2.A.4) gives

$$-\hat{p}^{k-2}\tilde{p}^{n-k}\,[\hat{p}\,(n-k-+2)+(k-1)\,\tilde{p}]<0.$$

As a result, the RHS of (2.A.3) is also decreasing in $\bar{x}$. If we set now $\bar{x}=\hat{p}$ in the RHS of (2.A.3), we obtain $-\tilde{p}^{n-k+1}\hat{p}^{k-1}\,(k+1)<0$. From this we conclude that (2.A.3) is negative. As a result, since the numerator of $\partial\eta_1(\tilde{p})\,/\partial\tilde{p}$ is positive and the denominator is negative, we infer that the function $\eta_1(\tilde{p})$ increases in $\tilde{p}$.[8]

Now consider the other equilibrium condition. Let us denote the LHS of (2.13)

[8] When $k=2$, equation (2.A.3) changes slightly. Therefore, we treat this case separately. If $k=2$ then

as $H(\hat{p}, \tilde{p})$. The condition $H(\hat{p}, \tilde{p}) = 0$ also defines an implicit relationship between $\hat{p}$ and $\tilde{p}$. Let the function $\eta_2(\tilde{p})$ define this relationship. This function is represented in Figure 2.A.1 below. By the implicit function theorem we have

$$\frac{\partial \eta_2(\tilde{p})}{\partial \tilde{p}} = -\frac{\partial H/\partial \tilde{p}}{\partial H/\partial \hat{p}}. \tag{2.A.6}$$

We note that H increases in $\hat{p}$. In fact,

$$\begin{aligned}\frac{\partial H}{\partial \hat{p}} &= (n-k)(\bar{x} - \hat{p} + \tilde{p})^{n-k-1}\left(1 - \bar{x}^k\right)\\ &\quad + (n-k)k\int_0^{\bar{x}-\hat{p}} (\varepsilon + \tilde{p})^{n-k-1}(\varepsilon + \hat{p})^{k-1}\,d\varepsilon > 0\end{aligned}$$

Moreover, H decreases in $\tilde{p}$. In fact, for $k < n-1$ we have

$$\frac{\partial H}{\partial \tilde{p}} = -\frac{1 - \bar{x}^{n-k}}{1 - \bar{x}} - (n-k)(\bar{x} - \hat{p} + \tilde{p})^{n-k-1}$$

$$g(\hat{p}, \tilde{p}) \equiv \frac{\int_0^{\overline{x}-\hat{p}} (\varepsilon + \tilde{p})^{n-2}(\varepsilon + 2\hat{p})\,d\varepsilon}{(\overline{x} - \hat{p} + \tilde{p})^{n-2}\,\overline{x}}$$

Then equation (2.A.3) is

$$\begin{aligned}&\bar{x}(\overline{x} - \hat{p} + \tilde{p})^{n-1}\left(\frac{\partial g}{\partial \hat{p}} - 1\right) = (n-2)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \tilde{p})^{n-2}(\varepsilon + 2\hat{p})\,d\varepsilon\\ &+ \frac{2}{n-1}(\bar{x} - \hat{p} + \tilde{p})^n - \frac{2}{n-1}\tilde{p}^{n-1}(\bar{x} - \hat{p} + \tilde{p}) - (2\bar{x} + \hat{p})(\overline{x} - \hat{p} + \tilde{p})^{n-1}\end{aligned} \tag{2.A.5}$$

The derivative of the RHS of (2.A.5) with respect to $\bar{x}$ is negative

$$\begin{aligned}&(n-2)(\bar{x} - \hat{p} + \tilde{p})^{n-2}(\bar{x} + \hat{p}) + \frac{2n}{n-1}(\bar{x} - \hat{p} + \tilde{p})^{n-1} - \frac{2}{n-1}\tilde{p}^{n-1}\\ &-2(\bar{x} - \hat{p} + \tilde{p})^{n-1} - (n-1)(2\bar{x} + \hat{p})(\bar{x} - \hat{p} + \tilde{p})^{n-2}\\ &= (\bar{x} - \hat{p} + \tilde{p})^{n-2}\left[(n-2)\bar{x} + (n-2)\hat{p} + \frac{2n}{n-1}\bar{x} - \frac{2n}{n-1}\hat{p}\right.\\ &+ \left.\frac{2n}{n-1}\tilde{p} - 2\bar{x} + 2\hat{p} - 2\tilde{p} - 2(n-1)\bar{x} - (n-1)\hat{p}\right] - \frac{2}{n-1}\tilde{p}^{n-1}\\ &= -(\bar{x} - \hat{p} + \tilde{p})^{n-2}\left[\bar{x}\frac{n^2-n-2}{n-1} + \hat{p}\frac{n+1}{n-1} - \frac{2}{n-1}\tilde{p}\right] - \frac{2}{n-1}\tilde{p}^{n-1}\\ &< -(\bar{x} - \hat{p} + \tilde{p})^{n-2}\left[\bar{x}\frac{n^2-n-2}{n-1} + \hat{p}\frac{n+1}{n-1} - \frac{2}{n-1}\bar{x}\right] - \frac{2}{n-1}\tilde{p}^{n-1}\\ &-(\bar{x} - \hat{p} + \tilde{p})^{n-2}\left[\bar{x}\frac{n^2-n-4}{n-1} + \hat{p}\frac{n+1}{n-1}\right] - \frac{2}{n-1}\tilde{p}^{n-1} < 0\end{aligned}$$

Since the expression is negative, the same arguments can be used to conclude that $\partial G/\partial \tilde{p}$ is positive also when $k = 2$, which implies that $\eta_1(\tilde{p})$ increases in $\tilde{p}$.

$$+ (n-k)(n-k-1)\int_0^{\bar{x}-\hat{p}} (\varepsilon+\tilde{p})^{n-k-2}(\varepsilon+\hat{p})^k d\varepsilon$$
$$< -\frac{1-\bar{x}^{n-k}}{1-\bar{x}} - (n-k)(\bar{x}-\hat{p}+\tilde{p})^{n-k-1}$$
$$+ (n-k)\bar{x}^k(n-k-1)\int_0^{\bar{x}-\hat{p}} (\varepsilon+\tilde{p})^{n-k-2} d\varepsilon$$
$$= -\frac{1-\bar{x}^{n-k}}{1-\bar{x}} - (n-k)(\bar{x}-\hat{p}+\tilde{p})^{n-k-1} + (n-k)\bar{x}^k(\bar{x}-\hat{p}+\tilde{p})^{n-k-1}$$
$$- (n-k)\bar{x}^k\tilde{p}^{n-k-1}$$
$$- \frac{1-\bar{x}^{n-k}}{1-\bar{x}} - (n-k)(\bar{x}-\hat{p}+\tilde{p})^{n-k-1}\left(1-\bar{x}^k\right) - (n-k)\bar{x}^k\tilde{p}^{n-k-1} < 0$$

while for $k = n-1$ we get $\partial H/\partial\tilde{p} = -2 < 0$. As a result, we conclude that the function η_2 is increasing in $\tilde{p}$.

Therefore, both η_1 and η_2 increase in $\tilde{p}$. To show that at least one pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ exists that satisfies the system of FOCs (2.12) and (2.13), we need to show that the functions η_1 and η_2 cross at least once in the space $[0;1/2] \times \left[0,\, p_k^m\right]$. As shown in Figure 2.A.1 we observe that $\eta_1(0) > 0$. For this, note that

$$G(\hat{p}, 0) = \frac{1-\bar{x}^k}{k\bar{x}^{k-1}} - \hat{p} + \frac{1}{\bar{x}^{k-1}(\bar{x}-\hat{p})^{n-k}}\int_0^{\bar{x}-\hat{p}} (\varepsilon+\hat{p})^{k-2}(\varepsilon+k\hat{p})\,\varepsilon^{n-k}d\varepsilon.$$

We have shown above that G decreases in $\hat{p}$. Therefore, since

$$G(0,0) = \frac{1-\bar{x}^k}{k\bar{x}^{k-1}} + \frac{1}{\bar{x}^{n-1}}\int_0^{\bar{x}} \varepsilon^{n-1}d\varepsilon > 0,$$

we conclude that $\eta_1(0) > 0$.

On the contrary, we now observe that $\eta_2(0) < 0$ (see Figure 2.A.1). This is because

$$H(\hat{p}, 0) = 1 - (\bar{x}-\hat{p})^{n-k} + (n-k)\int_0^{\bar{x}-\hat{p}} \varepsilon^{n-k-1}(\varepsilon+\hat{p})^k d\varepsilon$$

and $H(0,0) = 1 - \bar{x}^{n-k} + (n-k)\int_0^{\bar{x}} \varepsilon^{n-1}d\varepsilon > 0$.

Secondly, as depicted in Figure 2.A.1, we show that $\eta_1(1/2) < p_k^m < \eta_2(1/2)$, which ensures that the functions η_1 and η_2 cross at least once in the area $[0;1/2] \times \left[0,\, p_k^m\right]$. To see that $\eta_2(1/2) > p_k^m$, we first calculate

$$H\left(p_k^m, \frac{1}{2}\right) = 1 - \frac{1-\bar{x}^{n-k}}{1-\bar{x}}\frac{1}{2} - \left(\bar{x} - p_k^m + \frac{1}{2}\right)^{n-k}$$

$$+ (n-k) \int_0^{\bar{x}-p_k^m} \left(\varepsilon + \frac{1}{2}\right)^{n-k-1} (\varepsilon + p_k^m)^k \, d\varepsilon.$$

Now we argue that $H\left(p_k^m, \frac{1}{2}\right) < 0$. For this we take the partial derivative of $H\left(p_k^m, \frac{1}{2}\right)$ with respect to $\bar{x}$ we get

$$-\frac{1-(n-k)\,\bar{x}^{n-k-1} + (n-k-1)\,\bar{x}^{n-k}}{2\,(1-\bar{x})^2} - (n-k)\left(\bar{x} - p_k^m + \frac{1}{2}\right)^{n-k-1}\left(1-\bar{x}^k\right) < 0,$$

where the inequality follows from noting that the expression $1-(n-k)\,\bar{x}^{n-k-1} + (n-k-1)\,\bar{x}^{n-k}$ decreases in $\bar{x}$ and therefore it is higher than we set $\bar{x} = 1$, that is, $1-(n-k)\,\bar{x}^{n-k-1} + (n-k-1)\,\bar{x}^{n-k} \geq 1-(n-k)+(n-k-1) = 0$. We then conclude that $H\left(p_k^m, \frac{1}{2}\right)$ is decreasing in $\bar{x}$.

Setting $\bar{x}$ equal to its lowest possible value we get

$$H\left(p_k^m, \frac{1}{2}\right)\Big|_{\bar{x}=p_k^m} = 1 - \frac{1}{2^{n-k}} - \frac{1-(p_k^m)^{n-k}}{2\left(1-p_k^m\right)} \tag{2.A.7}$$

This expression is decreasing in n. In fact, its derivative with respect to n can be written as

$$\begin{aligned}
&\frac{2^{n-k-1}(p_k^m)^{n-k}\ln p_k^m + \left(1-p_k^m\right)\ln 2}{2^{n-k}\left(1-p_k^m\right)} \\
&< \frac{1}{2^{n-k}\left(1-p_k^m\right)}\left[p_k^m \ln p_k^m \left.(2p_k^m)^{n-k-1}\right|_{n=k+1} + (1-p_k^m)\ln 2\right] \\
&= \frac{1}{2^{n-k}\left(1-p_k^m\right)}\left[p_k^m \ln p_k^m + (1-p_k^m)\ln 2\right] < 0
\end{aligned}$$

The last inequality follows from the fact that $p_k^m \ln p_k^m + \left(1-p_k^m\right)\ln 2 < 0$. This can be shown in three steps. We check the sign of the expression with the lowest and highest values of k:

$$p_k^m \ln p_k^m + (1-p_k^m)\ln 2|_{k=2} = \frac{1}{\sqrt{3}}\ln\frac{1}{\sqrt{3}} + \left(1-\frac{1}{\sqrt{3}}\right)\ln 2 < 0$$

$$\lim_{k\to\infty}\left(p_k^m \ln p_k^m + (1-p_k^m)\ln 2\right) = 0$$

Let us now take the derivative of $p_k^m \ln p_k^m + \left(1-p_k^m\right)\ln 2$ with respect to k. We obtain the following $(\partial p_k^m/\partial k)\left(\ln p_k^m + 1 - \ln 2\right)$. The sign of this expression depends on the sign of $1-\ln 2 + \ln p_k^m = 1 - \ln 2 + \frac{1}{k}\ln(1+k)$. This expression is monotonically increasing in k, first negative and then positive. As a result, $p_k^m \ln p_k^m + (1 - p_k^m) \ln 2$ first decreases and then increases. Together with the two facts above, we conclude it is always negative.

Since $H\left(p_k^m, \frac{1}{2}\right)\Big|_{\bar{x}=p_k^m}$ is decreasing in n, if we set n equal to its lowest possible value, $k+1$, in (2.A.7) we obtain

$$\begin{aligned}
H\left(p_k^m, \frac{1}{2}\right)\Bigg|_{\bar{x}=p_k^m} \leq\ & H\left(p_k^m, \frac{1}{2}\right)\Bigg|_{\bar{x}=p_k^m; n=k+1} \\
=&1 - \frac{1}{2^{k+1-k}} - \frac{1 - {p_k^m}^{k+1-k}}{2\left(1 - p_k^m\right)} = 1 - \frac{1}{2} - \frac{1}{2} = 0
\end{aligned}$$

Therefore, since $H\left(p_k^m, \frac{1}{2}\right)$ is decreasing in $\bar{x}$, we conclude that $H\left(p_k^m, \frac{1}{2}\right)$ is always negative. And because H is increasing in $\hat{p}$, we obtain the result that $\eta_2\left(1/2\right) > p_k^m$.

We now show that $\eta_1\left(1/2\right) < p_k^m$. Since G is decreasing in $\hat{p}$, it suffices to demonstrate that

$$G\left(p_k^m, \frac{1}{2}\right) = \frac{1 - \overline{x}^k}{k\overline{x}^{k-1}} - \hat{p} + \frac{\int_0^{\overline{x}-\hat{p}} \left(\varepsilon + \hat{p}\right)^{k-2} \left(\varepsilon + \frac{1}{2}\right)^{n-k} \left(\varepsilon + k\hat{p}\right) d\varepsilon}{\left(\overline{x} - \hat{p} + \frac{1}{2}\right)^{n-k} \overline{x}^{k-1}} < 0.$$

Taking the derivative of $G\left(p_k^m, \frac{1}{2}\right)$ with respect to n gives

$$\begin{aligned}
&\left(\bar{x} - p_k^m + \frac{1}{2}\right)^{n-k} \bar{x}^{k-1} \frac{\partial G\left(p_k^m, \frac{1}{2}\right)}{\partial n} \\
&= \int_0^{\bar{x}-p_k^m} \left(\varepsilon + \frac{1}{2}\right)^{n-k} \left(\varepsilon + p_k^m\right)^{k-2} \left(\varepsilon + kp_k^m\right) \ln\left(\frac{\varepsilon + 1/2}{\bar{x} - p_k^m + 1/2}\right) d\varepsilon < 0
\end{aligned}$$

Since $G\left(p_k^m, \frac{1}{2}\right)$ decreases in n, we can set n equal to its lowest value and write

$$\begin{aligned}
&G\left(p_k^m, \frac{1}{2}\right) < G\left(p_k^m, \frac{1}{2}\right)\Bigg|_{n=k+1} \\
&= \frac{1 - \bar{x}^k}{k\bar{x}^{k-1}} - p_k^m + \frac{1}{\bar{x}^{k-1}\left(\bar{x} - p_k^m + \frac{1}{2}\right)} \int_0^{\bar{x}-p_k^m} \left(\varepsilon + p_k^m\right)^{k-2} \left(\varepsilon + \frac{1}{2}\right) \left(\varepsilon + kp_k^m\right) d\varepsilon \\
&= \frac{1}{k\bar{x}^{k-1}\left(\bar{x} - p_k^m + \frac{1}{2}\right)} T\left(\bar{x}\right)
\end{aligned}$$

where

$$T(\bar{x}) = \left(\bar{x} - p_k^m + \frac{1}{2}\right)\left(1 - \bar{x}^k - k\bar{x}^{k-1}p_k^m\right)$$
$$+ k\int_0^{\bar{x}-p_k^m} (\varepsilon + p_k^m)^{k-2}\left(\varepsilon + \frac{1}{2}\right)(\varepsilon + kp_k^m)\, d\varepsilon$$

Note that $\frac{1}{k\bar{x}^{k-1}\left(\bar{x}-p_k^m+\frac{1}{2}\right)} > 0$. Thus $G\left(p_k^m, \frac{1}{2}\right)\Big|_{n=k+1}$ is negative if $T(\bar{x}) < 0$. $T(\bar{x})$ decreases in $\bar{x}$ because

$$\frac{\partial T(\bar{x})}{\partial \bar{x}} = 1 - \bar{x}^k - k\bar{x}^{k-1}p_k^m - \left(\bar{x} - p_k^m + \frac{1}{2}\right)\left(k\bar{x}^{k-1} + k(k-1)\bar{x}^{k-2}p_k^m\right)$$
$$+ k\bar{x}^{k-2}\left(\bar{x} - p_k^m + \frac{1}{2}\right)(\bar{x} + (k-1)p_k^m)$$
$$= 1 - \bar{x}^k - k\bar{x}^{k-1}p_k^m$$

and this expression decreases in $\bar{x}$. Therefore, using $\bar{x} = p_k^m$, we can write

$$\frac{\partial T(\bar{x})}{\partial \bar{x}} < \frac{\partial T(\bar{x})}{\partial \bar{x}}\bigg|_{\bar{x}=p_k^m} = 1 - (p_k^m)^k - k(p_k^m)^{k-1}p_k^m = 0$$

Since $T(\bar{x})$ decreases in $\bar{x}$ we then conclude that $T(\bar{x}) < T\left(p_k^m\right) = 0$. As a result, the functions η_1 and η_2 cross at least once in the area $[0; 1/2] \times \left[0; p_k^m\right]$. *QED*

Claim 2.A.2. *The pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies (2.12) and (2.13) is unique.*

Proof. To show this uniqueness result, it is enough to show that η_1 increases in $\tilde{p}$ at a rate less than 1, while η_2 does so at a rate greater than 1. From (2.A.2), since $\partial G/\partial \hat{p} < 0$, we know that η_1 increases in $\tilde{p}$ if and only if $\partial G/\partial \hat{p} + \partial G/\partial \tilde{p} < 0$. For the case $k > 2$, we can then write

$$\overline{x}^{k-1}(\overline{x} - \hat{p} + \tilde{p})^{n-k-1}\left[\frac{\partial G(\hat{p},\tilde{p})}{\partial \tilde{p}} + \frac{\partial G(\hat{p},\tilde{p})}{\partial \hat{p}}\right]$$
$$= (n-k)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \hat{p})^{k-2}(\varepsilon + k\hat{p})(\varepsilon + \tilde{p})^{n-k-1}(\overline{x} - \hat{p} - \varepsilon)\, d\varepsilon$$
$$+ (n-k)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \hat{p})^{k-2}(\varepsilon + \tilde{p})^{n-k}(\varepsilon + k\hat{p})\, d\varepsilon$$
$$+ (\overline{x} - \hat{p} + \tilde{p})(k-1)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \hat{p})^{k-3}(\varepsilon + \tilde{p})^{n-k}(2\varepsilon + k\hat{p})\, d\varepsilon$$
$$- (\overline{x} - \hat{p} + \tilde{p})^{n-k}\,\overline{x}^{k-2}\left[2\overline{x} + (k-1)\hat{p}\right]$$

which can be simplified to

$$
\begin{aligned}
&\overline{x}^{k-1}\left(\overline{x}-\hat{p}+\tilde{p}\right)^{n-k-2}\left[\frac{\partial G\left(\hat{p},\tilde{p}\right)}{\partial\tilde{p}}+\frac{\partial G\left(\hat{p},\tilde{p}\right)}{\partial\hat{p}}\right]\\
&=\left(n-k\right)\int_0^{\overline{x}-\hat{p}}\left(\varepsilon+\hat{p}\right)^{k-2}\left(\varepsilon+k\hat{p}\right)\left(\varepsilon+\tilde{p}\right)^{n-k-1}d\varepsilon\\
&+\left(k-1\right)\int_0^{\overline{x}-\hat{p}}\left(\varepsilon+\hat{p}\right)^{k-3}\left(\varepsilon+\tilde{p}\right)^{n-k}\left(2\varepsilon+k\hat{p}\right)d\varepsilon\\
&-\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k}\bar{x}^{k-2}\left[2\bar{x}+\left(k-1\right)\hat{p}\right]
\end{aligned}
\tag{2.A.8}
$$

We now notice that the RHS of (2.A.8) decreases in $\bar{x}$. In fact, its derivative is

$$
\begin{aligned}
&\left(n-k\right)\bar{x}^{k-2}\left[\bar{x}+\left(k-1\right)\hat{p}\right]\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k-1}\\
&+\left(k-1\right)\bar{x}^{k-3}\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k}\left[2\bar{x}+\left(k-2\right)\hat{p}\right]\\
&-\left[\left(n-k\right)\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k-1}\bar{x}^{k-2}+\left(k-2\right)\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k}\bar{x}^{k-3}\right]\\
&\cdot\left[2\bar{x}+\left(k-1\right)\hat{p}\right]-2\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k}\bar{x}^{k-2}
\end{aligned}
$$

which, after rearranging, is equal to

$$
-(n-k)\bar{x}^{k-1}\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k-1}<0
$$

Therefore if (2.A.8) is negative when setting $\bar{x}=p_k^m$, then it is always negative, that is[9]

$$
\overline{x}^{k-1}\left(\overline{x}-\hat{p}+\tilde{p}\right)^{n-k}\left[\frac{\partial G\left(\hat{p},\tilde{p}\right)}{\partial\tilde{p}}+\frac{\partial G\left(\hat{p},\tilde{p}\right)}{\partial\hat{p}}\right]<-\tilde{p}^{n-k}\hat{p}^{k-1}\left(k+1\right)<0.
$$

[9] The same holds for the case when $k=2$. We have

$$
\begin{aligned}
&\overline{x}\left(\overline{x}-\hat{p}+\tilde{p}\right)^{n-2}\left[\frac{\partial G\left(\hat{p},\tilde{p}\right)}{\partial\tilde{p}}+\frac{\partial G\left(\hat{p},\tilde{p}\right)}{\partial\hat{p}}\right]=\left(n-2\right)\int_0^{\overline{x}-\hat{p}}\left(\varepsilon+\tilde{p}\right)^{n-3}\left(\varepsilon+2\hat{p}\right)d\varepsilon\\
&+\frac{2}{n-1}\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-1}-\frac{2}{n-1}\tilde{p}^{n-1}-\left(2\bar{x}+\hat{p}\right)\left(\overline{x}-\hat{p}+\tilde{p}\right)^{n-2}
\end{aligned}
\tag{2.A.9}
$$

The derivative of (2.A.9) with respect to $\bar{x}$ is negative because

$$
\begin{aligned}
&\left(n-2\right)\left(\bar{x}+\hat{p}\right)\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-3}+2\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-2}\\
&-2\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-2}-\left(n-2\right)\left(2\bar{x}+\hat{p}\right)\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-3}\\
&=-\bar{x}\left(n-2\right)\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-3}<0
\end{aligned}
$$

Then

$$
\overline{x}\left(\overline{x}-\hat{p}+\tilde{p}\right)^{n-2}\left(\frac{\partial G\left(\hat{p},\tilde{p}\right)}{\partial\tilde{p}}+\frac{\partial G\left(\hat{p},\tilde{p}\right)}{\partial\hat{p}}\right)<-3\hat{p}\tilde{p}^{n-2}<0
$$

Similarly, using (2.A.6), since $\partial H/\partial \hat{p} > 0$, we know that $\partial \eta_2/\partial \tilde{p} > 1$ if and only if $\partial H/\partial \hat{p} + \partial H/\partial \tilde{p} < 0$. For the case $k < n-1$, we then compute

$$\begin{aligned}
\frac{\partial H}{\partial \tilde{p}} + \frac{\partial H}{\partial \hat{p}} &= \frac{1-\bar{x}^{n-k}}{1-\bar{x}} - (n-k)\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k-1} \\
&\quad + (n-k)(n-k-1)\int_0^{\bar{x}-\hat{p}} (\varepsilon+\tilde{p})^{n-k-2}(\varepsilon+\hat{p})^k \, d\varepsilon \qquad (2.A.10) \\
&\quad + (n-k)\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k-1}\left(1-\bar{x}^k\right) \\
&\quad + (n-k)\,k\int_0^{\bar{x}-\hat{p}} (\varepsilon+\tilde{p})^{n-k-1}(\varepsilon+\hat{p})^{k-1} \, d\varepsilon \\
&= \frac{1-\bar{x}^{n-k}}{1-\bar{x}} - (n-k)\left(\bar{x}-\hat{p}+\tilde{p}\right)^{n-k-1}\bar{x}^k \\
&\quad + (n-k)(n-k-1)\int_0^{\bar{x}-\hat{p}} (\varepsilon+\tilde{p})^{n-k-2}(\varepsilon+\hat{p})^k \, d\varepsilon \\
&\quad + (n-k)\,k\int_0^{\bar{x}-\hat{p}} (\varepsilon+\tilde{p})^{n-k-1}(\varepsilon+\hat{p})^{k-1} \, d\varepsilon \qquad (2.A.11)
\end{aligned}$$

This expression decreases in $\bar{x}$ because its partial derivative with respect to $\bar{x}$, after rearranging, is equal to

$$-\frac{1-(n-k)\,\bar{x}^{n-k-1}+(n-k-1)\,\bar{x}^{n-k}}{(1-\bar{x})^2} < 0.$$

and we have already shown above that the numerator of this expression is positive. Thus using $\bar{x} = \hat{p}$ in (2.A.11) we can write[10]

$$\frac{\partial H}{\partial \tilde{p}} + \frac{\partial H}{\partial \hat{p}} < -\frac{1-\hat{p}^{n-k}}{1-\hat{p}} - (n-k)\,\tilde{p}^{n-k-1}\hat{p}^k < 0.$$

The result follows. *QED*.

Claim 2.A.3. *The price charged by the merging stores is higher than the price of the non-merging ones, that is,* $\hat{p}^* > \tilde{p}^*$.

Proof. Let $\tilde{p}_1$ be the price at which the function η_1 crosses the 45 degrees line, i.e., $\eta_1(\tilde{p}_1) = \tilde{p}_1$; likewise, let $\tilde{p}_2$ be such that $\eta_2(\tilde{p}_2) = \tilde{p}_2$ ($\tilde{p}_1$ and $\tilde{p}_2$ are represented in

[10] If $k = n-1$ then

$$\frac{\partial H}{\partial \tilde{p}} + \frac{\partial H}{\partial \hat{p}} = -2 + \left(1-\bar{x}^{n-1}\right) + (n-1)\int_0^{\bar{x}-\hat{p}} (\varepsilon+\hat{p})^{n-2}\, d\varepsilon = -1-\hat{p}^{n-1} < 0$$

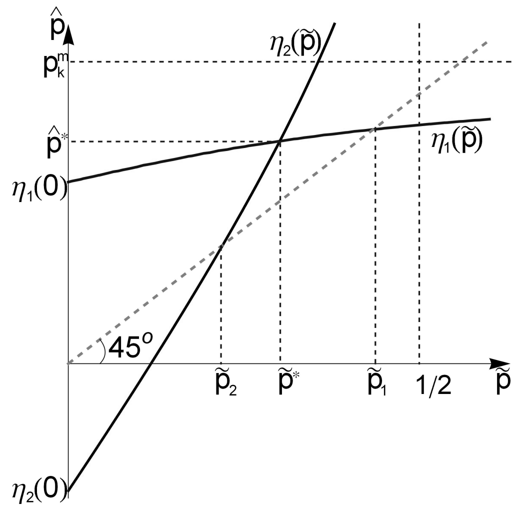


Figure 2.A.1. Existence and uniqueness of symmetric equilibrium

Figure 2.A.1). Given the properties of η_1 and η_2, if we show that $\tilde{p}_1 > \tilde{p}_2$ then we can conclude that $\hat{p}^* > \tilde{p}^*$.

When $\tilde{p} = \tilde{p}_1$ from the FOC $G(\eta_1(\tilde{p}), \tilde{p}) = 0$ we have

$$\tilde{p}_1 = \frac{1 - \overline{x}^k}{k\overline{x}^{k-1}} + \frac{\int_0^{\overline{x}-\tilde{p}_1} (\varepsilon + \tilde{p}_1)^{n-2} (\varepsilon + k\tilde{p}_1)\, d\varepsilon}{\overline{x}^{n-1}} \tag{2.A.12}$$

Similarly, when $\tilde{p} = \tilde{p}_2$ the FOC $H(\eta_2(\tilde{p}), \tilde{p}) = 0$ gives

$$\tilde{p}_2 = 1 - \bar{x} + \frac{1 - \bar{x}}{1 - \bar{x}^{n-k}} (n-k) \int_0^{\bar{x}-\tilde{p}_2} (\varepsilon + \tilde{p}_2)^{n-1}\, d\varepsilon \tag{2.A.13}$$

For a contradiction, suppose $\tilde{p}_2 > \tilde{p}_1$. Then the difference between the RHS of (2.A.12) and the RHS of (2.A.13) must be negative. Let us denote this difference V and note that

$$\begin{aligned} V &\equiv \frac{1 - \bar{x}^k}{k\bar{x}^{k-1}} + \frac{\int_0^{\bar{x}-\tilde{p}_1} (\varepsilon + \tilde{p}_1)^{n-2} (\varepsilon + k\tilde{p}_1)\, d\varepsilon}{\overline{x}^{n-1}} - 1 + \bar{x} \\ &\quad - \frac{1 - \bar{x}}{1 - \bar{x}^{n-k}} (n-k) \int_0^{\bar{x}-\tilde{p}_2} (\varepsilon + \tilde{p}_2)^{n-1}\, d\varepsilon \\ &= \frac{\int_0^{\bar{x}-\tilde{p}_1} (\varepsilon + \tilde{p}_1)^{n-2} (\varepsilon + k\tilde{p}_1)\, d\varepsilon}{\overline{x}^{n-1}} + \frac{1 + (k-1)\bar{x}^k - k\bar{x}^{k-1}}{k\bar{x}^{k-1}} \end{aligned}$$

$$-\frac{1-\bar{x}}{1-\bar{x}^{n-k}}\left(n-k\right)\int_0^{\bar{x}-\tilde{p}_2}\left(\varepsilon+\tilde{p}_2\right)^{n-1}d\varepsilon$$
$$>\frac{\int_0^{\overline{x}-\tilde{p}_1}\left(\varepsilon+\tilde{p}_1\right)^{n-1}d\varepsilon}{\overline{x}^{n-1}}+\frac{1+\left(k-1\right)\overline{x}^k-k\bar{x}^{k-1}}{k\overline{x}^{k-1}}$$
$$-\frac{1-\bar{x}}{1-\bar{x}^{n-k}}\left(n-k\right)\int_0^{\bar{x}-\tilde{p}_1}\left(\varepsilon+\tilde{p}_1\right)^{n-1}d\varepsilon \tag{2.A.14}$$

where the inequality follows from replacing $\varepsilon+k\tilde{p}_1$ by $\varepsilon+\tilde{p}_1$ in the first integral.

Since the second integral in (2.A.14) is equal to $\left[\bar{x}^n-\left(\tilde{p}_1\right)^n\right]$, the whole expression in (2.A.14) increases in $\tilde{p}_2$. Therefore, (2.A.14) must be higher than when we replace $\tilde{p}_2$ by $\tilde{p}_1$. That is, (2.A.14) is higher than

$$\frac{1}{\overline{x}^{n-1}}\int_0^{\overline{x}-\tilde{p}_1}\left(\varepsilon+\tilde{p}_1\right)^{n-1}d\varepsilon+\frac{1+\left(k-1\right)\overline{x}^k-k\bar{x}^{k-1}}{k\overline{x}^{k-1}}$$
$$-\frac{1-\bar{x}}{1-\bar{x}^{n-k}}\left(n-k\right)\int_0^{\bar{x}-\tilde{p}_1}\left(\varepsilon+\tilde{p}_1\right)^{n-1}d\varepsilon$$
$$=\frac{\bar{x}^n-\tilde{p}_1^n}{n}\left[\frac{1}{\bar{x}^{n-1}}-\frac{\left(1-\bar{x}\right)\left(n-k\right)}{1-\bar{x}^{n-k}}\right]+\frac{1+\left(k-1\right)\overline{x}^k-k\bar{x}^{k-1}}{k\overline{x}^{k-1}}$$
$$=\frac{\bar{x}^n-\tilde{p}_1^n}{n\left(1-\bar{x}^{n-k}\right)}\left[\frac{1-\bar{x}^{n-k}-\left(n-k\right)\bar{x}^{n-1}\left(1-\bar{x}\right)}{\bar{x}^{n-1}}\right]$$
$$+\frac{1+\left(k-1\right)\overline{x}^k-k\bar{x}^{k-1}}{k\overline{x}^{k-1}} \tag{2.A.15}$$

We now show that this last expression is positive, which establishes a contradiction. For this, we first note that the term in square brackets is positive. To see this, we take the derivative with respect to $\bar{x}$ which gives

$$n-k+\left(k+1\right)\bar{x}^{-}k-\left(n-1\right)\bar{x}^{-n}$$

and note that this expression is decreasing in n (its derivative is $-\bar{x}^{-n}\left[1-\bar{x}^n-\left(n-1\right)\ln\bar{x}\right]<0$). Then we can et $n=k+1$ and write that $n-k+\left(k+1\right)\bar{x}^{-}k-\left(n-1\right)\bar{x}^{-n}\leq 1+\left(k+1\right)\bar{x}^{-}k-\left(n-1\right)\bar{x}^{-(k+1)}$. This last expression increases in $\bar{x}$ (its derivative is $k\bar{x}^{-(k+2)}\left[1+\bar{x}+k\left(1+\bar{x}\right)\right]>0$) and therefore we can write $1+\left(k+1\right)\bar{x}^{-k}-k\bar{x}^{-(k+1)}<\leq 0$. Therefore, the square bracket decreases in $\bar{x}$ so we can use the value $\bar{x}=1$ and show that

$$\frac{1-\bar{x}^{n-k}-\left(n-k\right)\bar{x}^{n-1}\left(1-\bar{x}\right)}{\bar{x}^{n-1}}>0$$

Finally, we observe that the last term of (2.A.15) is also positive. This follows from the fact that the numerator $1+(k+1)\,\bar{x}^k - k\bar{x}^{k+1}$ decreases in $\bar{x}$ and then we can use the value $\bar{x}=1$ to write $1+(k+1)\,\bar{x}^k - k\bar{x}^{k+1} > 0$.

Therefore, we have proven that $V > 0$, which is impossible if $\tilde{p}_2 > \tilde{p}_1$. As a result, $\hat{p}^* > \tilde{p}^*$. *QED* ■.

Proof of Proposition 2.2. Since we have already proven that $\hat{p}^* > \tilde{p}^*$, we focus on the inequality $\tilde{p}^* > p^*$. Furthermore, we claim that $\hat{p}^* > p^*$. According to Zhou (2009) proposition 3, the price of non-prominent firms, because of the search order effect, is always higher than the equilibrium price when firms are searched randomly. The price of a merger increases due to the order effect, because a merger is visited only all non-merged shops are searched, and due to price internalization effect. Therefore, $\hat{p}^* > p^*$.

(a) Consider the case in which the search cost is sufficiently low, that is, $\bar{x} \to 1$. We prove $\tilde{p}^* > p^*$ by contradiction. Therefore, assume that, on the contrary, $\tilde{p}^* < p^*$ when $\bar{x} \to 1$. The aggregate quantity sold in the market by all firms together is $Q = 1 - \hat{p}^{*k}\tilde{p}^{*n-k}$. Using the FOCs and denoting the quantity sold by a non-merged firm by firm by $\tilde{q}^*$, and that sold by all the merging firms together by $\hat{q}^*$ we can state that

$$Y\left(\tilde{p}^*, \hat{p}^*\right) \equiv Q - (n-k)\tilde{q}^* - \hat{q}^* = 0$$

where

$$(n-k)\tilde{q}^* = \frac{1-\overline{x}^{n-k}}{1-\overline{x}}\tilde{p}^*$$

$$\hat{q}^* = k\hat{p}^*\overline{x}^{k-1}\left(\overline{x}-\hat{p}^*+\tilde{p}^*\right)^{n-k} - k\left(k-1\right)\hat{p}^*\int_0^{\overline{x}-\hat{p}^*}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+\tilde{p}^*\right)^{n-k}d\varepsilon.$$

We now argue that $Y\left(\tilde{p}^*, \hat{p}^*\right)$ is decreasing in $\tilde{p}^*$. This is because $\partial Q/\partial\tilde{p}^* < 0$, $\partial\tilde{q}^*/\partial\tilde{p}^* > 0$ and

$$\begin{aligned}
\frac{1}{k(n-k)\hat{p}^*}\frac{\partial\hat{q}^*}{\partial\tilde{p}^*} &= \overline{x}^{k-1}\left(\overline{x}-\hat{p}^*+\tilde{p}^*\right)^{n-k-1} \\
&- (k-1)\int_0^{\overline{x}-\hat{p}^*}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+\tilde{p}^*\right)^{n-k-1}d\varepsilon \\
&> \overline{x}^{k-1}\left(\overline{x}-\hat{p}^*+\tilde{p}^*\right)^{n-k-1} \\
&- \left(\overline{x}-\hat{p}^*+\tilde{p}^*\right)^{n-k-1}(k-1)\int_0^{\overline{x}-\hat{p}^*}\left(\varepsilon+\hat{p}^*\right)^{k-2}d\varepsilon \\
&= \left(\overline{x}-\hat{p}^*+\tilde{p}^*\right)^{n-k-1}\hat{p}^{*k-1} > 0
\end{aligned}$$

Next, since Y is decreasing in $\tilde{p}^*$ and by assumption $\tilde{p}^* < p^*$ we must have $Y(p^*, \hat{p}^*) < 0$. When $\overline{x} \to 1$, $Y(p^*, \hat{p}^*)$ goes to:

$$1 - \hat{p}^{*k} p^{*n-k} - (n-k)\, p^* - k\hat{p}^* \left(1 - \hat{p}^* + p^*\right)^{n-k} + k(k-1)\,\hat{p}^* \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-2} \left(\varepsilon + p^*\right)^{n-k} d\varepsilon \tag{2.A.16}$$

Now we invoke the FOC of the merged entity, denoted above by $G(\hat{p}^*, \tilde{p}^*)$. The function $G(\hat{p}^*, \tilde{p}^*)$ was shown to be increasing in $\tilde{p}^*$ so when $\tilde{p}^* < p^*$ we must have $G(\hat{p}^*, p^*) > G(\hat{p}^*, \tilde{p}^*) = 0$. Therefore:

$$\begin{aligned}\lim_{\bar{x}\to 1} G(\hat{p}^*, p^*) &\equiv -\hat{p}^* + \frac{1}{\left(1 - \hat{p}^* + p^*\right)^{n-k}} \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-2} \left(\varepsilon + p^*\right)^{n-k} \left(\varepsilon + k\hat{p}\right) d\varepsilon \\ &= -\hat{p}^* + \frac{k-1}{\left(1 - \hat{p}^* + p^*\right)^{n-k}}\,\hat{p}^* \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-2} \left(\varepsilon + p^*\right)^{n-k} d\varepsilon \\ &+ \frac{1}{\left(1 - \hat{p}^* + p^*\right)^{n-k}} \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-1} \left(\varepsilon + p^*\right)^{n-k} d\varepsilon\end{aligned}$$

must be positive, which implies that it must be the case that

$$\begin{aligned}&- k\hat{p}^* \left(1 - \hat{p}^* + p^*\right)^{n-k} + k(k-1)\,\hat{p}^* \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-2} \left(\varepsilon + p^*\right)^{n-k} d\varepsilon \\ &> -k \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-1} \left(\varepsilon + p^*\right)^{n-k} d\varepsilon\end{aligned}$$

Using this inequality in (2.A.16), we get that

$$\lim_{\bar{x}\to 1} Y(p^*, \hat{p}^*) > 1 - \hat{p}^{*k} p^{*n-k} - (n-k)\, p^* - k \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-1} \left(\varepsilon + p^*\right)^{n-k} d\varepsilon \tag{2.A.17}$$

This last expression is increasing in $\hat{p}$. This is because its derivative with respect to $\hat{p}^*$ is the same as the sign of the following expression

$$\begin{aligned}&- \hat{p}^{*k-1} p^{*n-k} - (k-1) \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-2} \left(\varepsilon + p^*\right)^{n-k} d\varepsilon + \left(1 - \hat{p}^* + p^*\right)^{n-k} \\ &> -\hat{p}^{*k-1} p^{*\,n-k} - (k-1)\left(1 - \hat{p}^* + p^*\right)^{n-k} \int_0^{1-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-2} d\varepsilon + \left(1 - \hat{p}^* + \tilde{p}^*\right)^{n-k} \\ &= \left(1 - \hat{p}^* + p^*\right)^{n-k} \hat{p}^{*k-1} - \hat{p}^{*k-1} p^{*n-k} > 0\end{aligned}$$

Therefore, (2.A.17) is greater than after setting $\hat{p}^* = p^*$, that is

$$\begin{aligned}\lim_{\bar{x}\to 1} Y\left(p^*, \hat{p}^*\right) &> 1 - \left(p^*\right)^n - (n-k)\,p^* - k\left[\frac{1}{n} - \frac{1}{n}\left(p^*\right)^n\right] \\ &> np^* - (n-k)\,p^* - kp^* = 0\end{aligned}$$

where for the last equality we have used the FOC of a firm in the pre-merger market. (If $\bar{x} \to 1$ then the first order condition of a firm in a pre-merger market becomes $1 - np^* - (p^*)^n = 0$.) Consequently, if $\tilde{p} > p^*$ then we have $\lim_{\bar{x}\to 1} Y\left(p^*, \hat{p}\right) > 0$, which establishes a contradiction.

(b) Consider the case in which the search cost is sufficiently high, that is, $\bar{x} \to p_k^m$. From proposition 2.1 we know that the solution to the FOCs is unique. Therefore, if we find two prices for which (2.12) and (2.13) hold when $\bar{x} \to p_k^m$ then these prices are indeed the equilibrium prices. Let us take the limit of the LHS of (2.12) and (2.13) when $\bar{x} \to p_k^m$. and let us use the notation $\lim_{\bar{x}\to p_k^m} \tilde{p}^* = \tilde{p}_l$ and $\lim_{\bar{x}\to p_k^m} \hat{p}^* = p_k^m$. Then we get the following expressions

$$\begin{aligned}\left(\tilde{p}^*\right)^{n-k}\left(1 - (k+1)\left(p_k^m\right)^k\right) &= 0 \\ \left(1 - p_k^m\right)\left(1 - \tilde{p}_l^{n-k}\right) - \tilde{p}_l\left(1 - \left(p_k^m\right)^{n-k}\right) &= 0 \qquad (2.A.18)\end{aligned}$$

The first equation is indeed zero, given the definition of p_k^m and the second equation therefore gives the value of $\tilde{p}_l$ when $\bar{x} \to p_k^m$. We note that the price $\tilde{p}_l$ is less than $p^m = 1/2$ because, as shown in the proof of proposition 2.1 $H\left(p_k^m, 1/2\right) \leq 0$. We are interested in comparing $\tilde{p}_l$ with the pre-merger equilibrium price. Let us use the notation $p_l = \lim_{\bar{x}\to p_k^m} p^*$. We now argue that $\tilde{p}_l > p_l$. To show this, we take the limit when $\bar{x} \to p_k^m$ of the FOC that determines p_l. This gives:

$$\left(1 - p_k^m\right)\left(1 - p_l^n\right) - p_l\left[1 - \left(p_k^m\right)^n\right] = 0$$

If we fix the value of p_k^m then the solution of this equation, p_l, decreases with n. Comparing this equation with (2.A.18), since $n - k < n$, it is immediately clear that $\tilde{p}_l > p_l$.

(c) Finally, we look at the case $n = 3$. If $n = 3$ then the FOC of a merging firm

$$\left(\bar{x} - \hat{p}^* + \tilde{p}^*\right)\left(1 - \bar{x}^2 - 2\hat{p}^*\bar{x}\right) + 2\int_0^{\bar{x}-\hat{p}^*} \left(\varepsilon + \tilde{p}^*\right)\left(\varepsilon + 2\hat{p}^*\right) d\varepsilon = 0$$

may be rearranged as follows

$$\hat{p}^{*3} - \hat{p}^{*}\bar{x}^{2} - \tilde{p}^{*}\left(3\hat{p}^{*2} - 1\right) = \frac{\bar{x}^{3}}{3} - \frac{\hat{p}^{*3}}{3} - \bar{x} + \hat{p}^{*} \tag{2.A.19}$$

The FOC of a non-merging firm

$$1 - 2\tilde{p}^{*} - \bar{x} + \hat{p}^{*} + \int_{0}^{\bar{x}-\hat{p}^{*}} (\varepsilon + \hat{p}^{*})^{2}\, d\varepsilon = 0$$

gives us the relation

$$\frac{\bar{x}^{3}}{3} - \frac{\hat{p}^{*3}}{3} - \bar{x} + \hat{p}^{*} = 2\tilde{p}^{*} - 1$$

Using this in (2.A.19) we have

$$\hat{p}^{*3} - \hat{p}^{*}\bar{x}^{2} - 3\tilde{p}^{*}\hat{p}^{*2} - \tilde{p}^{*} + 1 = 0$$

or

$$\tilde{p}^{*} = \frac{1 + \hat{p}^{*3} - \hat{p}^{*}\bar{x}^{2}}{1 + 3\hat{p}^{*2}}$$

From the FOC (2.4) in the pre-merger market we get that

$$p^{*} = \frac{1 - p^{*3}}{1 + \bar{x} + \bar{x}^{2}}$$

Since, by strategic complementarity, $\tilde{p}^{*}$ increases in $\hat{p}^{*}$ and since $\hat{p}^{*} > p^{*}$, the difference $\tilde{p}^{*} - p^{*}$ is greater than when we replace $\hat{p}^{*}$ by p^{*}. Therefore,

$$\begin{aligned} \tilde{p}^{*} - p^{*} &= \frac{1 + \hat{p}^{*3} - \hat{p}^{*}\bar{x}^{2}}{1 + 3\hat{p}^{*2}} - \frac{1 - p^{*3}}{1 + \bar{x} + \bar{x}^{2}} \\ &> \frac{1 + p^{*3} - p^{*}\bar{x}^{2}}{1 + 3p^{*2}} - \frac{1 - p^{*3}}{1 + \bar{x} + \bar{x}^{2}} \end{aligned} \tag{2.A.20}$$

The RHS of this expression is concave in $\bar{x}$ because its second derivative with respect to $\bar{x}$ is negative:

$$-\frac{2p^{*}}{1 + 3p^{*2}} + \frac{6\left(p^{*3} - 1\right)\bar{x}(1 + \bar{x})}{\left(1 + \bar{x} + \bar{x}^{2}\right)^{3}} < 0$$

Hence, if the RHS of (2.A.20) is positive with the highest and the lowest possible values of $\bar{x}$ then it is positive for all $\bar{x}$ values. Setting $\bar{x} = 1$ in the RHS of (2.A.20) gives

$$\frac{2 - 3p^* - 3p^{*2} + 4p^{*3} + 3p^{*5}}{3\left(1 + 3p^{*2}\right)} \tag{2.A.21}$$

which is always positive as shown in Figure 2.A.2.

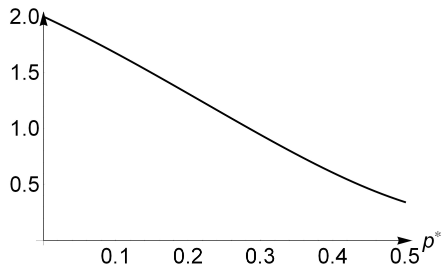


Figure 2.A.2. Plot of expression 2.A.21

Setting $\bar{x} = p^*$ in (2.A.20) gives

$$\frac{p\left(1 - 3p^* + 3p^{*2}\right)}{1 + 3p^{*2}} > 0.$$

Thus, $\tilde{p}^* > p^*$ ■.

Proof of Proposition 2.3. We first prove that the function $\eta_1\left(\tilde{p}\right)$ shifts downwards if $\bar{x}$ increases. Form relation $G\left(\eta_1\left(\tilde{p}\right), \tilde{p}, \bar{x}\right) = 0$ we obtain that $\partial\eta_1\left(\tilde{p}\right)/\partial\bar{x} = -\left(\partial G/\partial\bar{x}\right)/\left(\partial G/\partial\hat{p}\right)$. Above the proof of Proposition 2.1, we have already shown that $\partial G/\partial\hat{p} < 0$. Therefore, we need to show that $\partial G/\partial\bar{x} < 0$ where

$$\begin{aligned}\frac{\partial G}{\partial \bar{x}} &= -\frac{(k-1)\left(1 - \overline{x}^k - k\hat{p}\overline{x}^{k-1}\right)}{k\overline{x}^k} \\ &\quad - \frac{(k-1)\left(\overline{x} - \hat{p} + \tilde{p}\right) + (n-k)\,\overline{x}}{\overline{x}^k\left(\overline{x} - \hat{p} + \tilde{p}\right)^{n-k+1}} \int_0^{\overline{x}-\hat{p}} \left(\varepsilon + \hat{p}\right)^{k-2}\left(\varepsilon + \tilde{p}\right)^{n-k}\left(\varepsilon + k\hat{p}\right) d\varepsilon\end{aligned}$$

Using the FOC $G\left(\hat{p}^*, \tilde{p}^*\right) = 0$ we can rewrite $\partial G/\partial\bar{x}$ as follows

$$\begin{aligned}\frac{\partial G}{\partial \bar{x}} &= -\frac{(k-1)\left(1 - \overline{x}^k - k\hat{p}\overline{x}^{k-1}\right)}{k\overline{x}^k} + \left[\frac{(k-1)\left(\overline{x} - \hat{p} + \tilde{p}\right) + (n-k)\,\overline{x}}{\overline{x}^k\left(\overline{x} - \hat{p} + \tilde{p}\right)^{n-k+1}}\right] \\ &\quad \cdot \frac{\left(\bar{x} - \hat{p} + \tilde{p}\right)^{n-k}\left(1 - \bar{x}^k - k\bar{x}^{k-1}\hat{p}\right)}{k}\end{aligned}$$

$$= -\frac{(k-1)\left(1-\overline{x}^k - k\hat{p}\overline{x}^{k-1}\right)}{k\overline{x}^k}\left[k-1-\frac{(k-1)(\overline{x}-\hat{p}+\tilde{p})+(n-k)\overline{x}}{(\overline{x}-\hat{p}+\tilde{p})}\right]$$
$$= \frac{(k-1)\left(1-\overline{x}^k - k\hat{p}\overline{x}^{k-1}\right)(n-k)}{k\overline{x}^{k-1}(\overline{x}-\hat{p}+\tilde{p})} < 0$$

where the last inequality follows from the fact that for $G(\hat{p},\tilde{p}) = 0$ it must be the case that $1-\bar{x}^k - k\bar{x}^{k-1}\hat{p} < 0$. As a result, we conclude that $\eta_1(\tilde{p})$ shifts downwards when $\bar{x}$ increases.

We now show that $\eta_2(\tilde{p})$ shifts upwards when $\bar{x}$ increases. From the relation $H(\eta_2(\tilde{p}),\tilde{p},\bar{x}) = 0$, we obtain that $\partial\eta_2(\tilde{p})/\partial\bar{x} = -(\partial H/\partial\bar{x})/(\partial H/\partial\hat{p})$. Above in the proof of proposition 2.1, we have already shown that $\partial H/\partial\tilde{p} < 0$. Therefore, we need to show that $\partial H/\partial\bar{x} < 0$. In fact, taking the derivative of H with respect to $\bar{x}$ gives

$$\frac{1}{n-k}\frac{\partial H}{\partial \bar{x}} = -\frac{\tilde{p}\left[1-(n-k)\overline{x}^{n-k-1}+(n-k-1)\overline{x}^{n-k}\right]}{(n-k)(1-\overline{x})^2} - (\overline{x}-\hat{p}+\tilde{p})^{n-k-1}\left(1-\overline{x}^k\right) < 0$$

where the last inequality uses the fact tat $1-(n-k)\bar{x}^{n-k-1}+(n-k-1)\bar{x}^{n-k}$ is decreasing in $\bar{x}$ and equals 0 when $\bar{x} = 1$.

Since $\eta_1(\tilde{p})$ shifts downwards while $\eta_2(\tilde{p})$ shifts upwards when $\bar{x}$ increases, we conclude that $\hat{p}^*$ and $\tilde{p}^*$ decrease in $\bar{x}$ (or increase in s). ■.

Proof of Proposition 2.4. Post-merger, the profit of a merging firm is $\hat{\pi}^*/k$, while pre-merger it is π^*. The we need to consider the difference $\hat{\pi}^*/k - \pi^*$.

(a) To prove this we set $k = 2$ in the profits difference $\hat{\pi}^*/k - \pi^*$ and study its sign when $\overline{x} \to p_k^m (= 1/\sqrt{3})$. Then, for $k = 2$, the profit of one merging firm is

$$\lim_{\overline{x}\to 1/\sqrt{3}} \frac{\hat{\pi}^*}{2} = \frac{\tilde{p}_l^{n-2}}{3\sqrt{3}}$$

where we use the same notation as a bove in the proof of proposition 2.2: $\tilde{p}_l = \lim_{\overline{x}\to p_k^m} \tilde{p}^*$. We have shown above that $\tilde{p}_l < p^m = 1/2$. Therefore,

$$\lim_{\overline{x}\to 1/\sqrt{3}} \frac{\hat{\pi}^*}{2} < \frac{(p^m)^{n-k}}{3\sqrt{3}}$$

which implies that

$$\lim_{\bar{x}\to 1/\sqrt{3}}\left[\frac{\hat{\pi}^*}{2}-\pi^*\right]<\frac{(1/2)^{n-2}}{3\sqrt{3}}-\frac{(p_l)^2\,(1-(p_2^m)^n)}{n(1-p_2^m)}$$

where, again as in the proof of proposition 2.2, $p_l = \lim \bar{x} \to p_k^m p^*$. If we demonstrate that

$$\frac{(p_l)^2\,(1-(p_2^m)^n)}{n(1-p_2^m)}>\frac{1}{2^{n-2}3\sqrt{3}}$$

or

$$p_l>\sqrt{\frac{n(1-p_2^m)}{2^{n-2}3\sqrt{3}(1-\left(p_2^m\right)^n)}}=\sqrt{\frac{n(1-3^{-1/2})}{2^{n-2}3\sqrt{3}(1-3^{-n/2})}} \tag{2.A.22}$$

then the result follows.

To show that (2.A.22) holds, we observe that the FOC

$$1-p^{*n}-p^*\frac{(1-\overline{x}^n)}{1-\overline{x}}=0 \tag{2.A.23}$$

which determines the value of p^*, is decreasing in p^*. Taking the limit of (2.A.23) when $\overline{x}\to 1/\sqrt{3}$ gives

$$1-(p_l)^n-p_l\frac{\left(1-3^{-n/2}\right)}{1-3^{-1/2}}=0$$

Now, using (2.A.22), if we replace p_l by $\left(\frac{n(1-3^{-1/2})}{2^{n-2}3\sqrt{3}(1-3^{-n/2})}\right)^{\frac{1}{2}}$ in this expression we get

$$1-\left[\frac{n(1-3^{-1/2})}{2^{n-2}3\sqrt{3}(1-3^{-n/2})}\right]^{\frac{n}{2}}-\sqrt{\frac{n(1-3^{-1/2})}{2^{n-2}3\sqrt{3}(1-3^{-n/2})}}\frac{\left(1-3^{-n/2}\right)}{1-3^{-1/2}} \tag{2.A.24}$$

which is always positive as shown in Figure 2.A.4.

Since (2.A.23) is decreasing in p^*, then (2.A.22) must hold.

(B) Using the definition of p_k^m, we have that $1-(p_k^m)^k = k(p_k^m)^k$. Therefore we can write

$$\lim_{\bar{x}\to p_k^m;n\to\infty}\left[\frac{\hat{\pi}^*}{k}-\pi^*\right]<\lim_{n\to\infty}\left[\frac{p_k^m}{2^{n-k}}\left(1-(p_k^m)^k\right)-\frac{(p_l)^2\left(1-\left(p_k^m\right)^n\right)}{n\left(1-p_k^m\right)}\right]$$

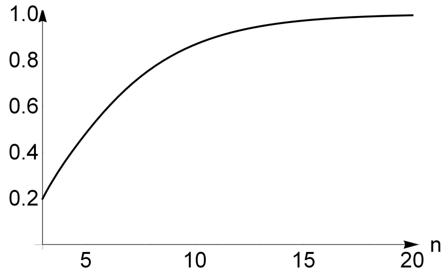


Figure 2.A.3. Plot of expression (2.A.24).

$$= \lim_{n\to\infty}\left[\frac{\left(p_k^m\right)^{k+1}}{2^{n-k}}-\frac{\left(p_l\right)^2\left(1-\left(p_k^m\right)^n\right)}{n\left(1-p_k^m\right)}\right]$$

Note that $1-\left(p_k^m\right)^n > 1-\left(p_k^m\right)^k = k\left(p_k^m\right)^k$. Thus

$$\lim_{n\to\infty}\left[\frac{\left(p_k^m\right)^{k+1}}{2^{n-k}}-\frac{\left(p_l\right)^2\left(1-\left(p_k^m\right)^n\right)}{n\left(1-p_k^m\right)}\right] < \lim_{n\to\infty}\left[\frac{\left(p_k^m\right)^{k+1}}{2^{n-k}}-\frac{\left(p_l\right)^2 k\left(p_k^m\right)^k}{n\left(1-p_k^m\right)}\right]$$
$$= \frac{\left(p_k^m\right)^k}{1-p_k^m}\lim_{n\to\infty}\left[\frac{p_k^m\left(1-p_k^m\right)}{2^{n-k}}-\frac{\left(p_l\right)^2 k}{n}\right] = 0$$

which shows that for any k, merging is not profitable whenever search costs and the number of competitors are sufficiently high.

(C) To prove this, we show that in the limit when $\bar{x}\to 1$ (or search costs go to zero), $\hat{\pi}^*/k-\pi^* > 0$. Notice that for this it suffices to show that

$$\lim_{\bar{x}\to 1}\left[\hat{\pi}^*/k-\frac{\tilde{p}^*}{n}\left(1-\tilde{p}^{*n}\right)\right] > 0,$$

because in the limit when $\bar{x}\to 1$, $\frac{\tilde{p}^*}{n}\left(1-\tilde{p}^{*n}\right) > \frac{p^*}{n}\left(1-p^{*n}\right)$.

Observe that equilibrium profits of the merging firms are

$$\hat{\pi}^* = \hat{p}^*\left[\left(\bar{x}-\hat{p}^*+\tilde{p}^*\right)^{n-k}\left(1-\bar{x}^k\right)+k\int_0^{\bar{x}-\hat{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k}\left(\varepsilon+\hat{p}^*\right)^{k-1}d\varepsilon\right]$$

Taking the derivative of $\hat{\pi}^*$ wth respect to $\hat{p}^*$ and taking the limit when $\bar{x} \to 1$ gives

$$-\left(1-\hat{p}^*+\tilde{p}^*\right)^{n-k} k\hat{p}^* + k\int_0^{1-\hat{p}^*} \left(\varepsilon+\tilde{p}^*\right)^{n-k} \left(\varepsilon+\hat{p}^*\right)^{k-2} \left(\varepsilon+k\hat{p}^*\right) d\varepsilon.$$

This expression is identical to the FOC (2.12) when $\bar{x} \to 1$. As a result, for any value of $\hat{p} < \hat{p}^*$ (where $\hat{p}^*$ is the equilibrium price of a merging firm when $\bar{x} \to 1$), this derivative is negative. Since the equilibrium price of a non-merging firm is lower than the price of a merging one, we have

$$\begin{aligned}
\lim_{\bar{x}\to 1} \hat{\pi}^* &= \lim_{\bar{x}\to 1} \left[\hat{p}^* k \int_0^{1-\hat{p}^*} \left(\varepsilon+\tilde{p}^*\right)^{n-k} \left(\varepsilon+\hat{p}^*\right)^{k-1} d\varepsilon\right] \\
&> \lim_{\bar{x}\to 1} \left[\tilde{p}^* k \int_0^{1-\tilde{p}^*} \left(\varepsilon+\tilde{p}^*\right)^{n-k} \left(\varepsilon+\tilde{p}^*\right)^{k-1} d\varepsilon\right] \\
&= \lim_{\bar{x}\to 1} \frac{k\tilde{p}^*}{n} \left(1-\tilde{p}^{*n}\right)
\end{aligned}$$

where the last inequality follows from the observations that $\tilde{p}^* \left(1-\tilde{p}^{*n}\right)$ increases in $\tilde{p}^*$ and $\tilde{p}^* > p^*$. The result the follows. ■.

Proof of Proposition 2.5. We start by deriving the payoffs of the firms in the situation where consumers start searching at the merging stores. As above, let $\hat{p}^*$ and $\tilde{p}^*$ denote the equilibrium prices of merging and non-merging stores, respectively. Consider first the payoff of the merging stores when they deviate by charging a price $\hat{p} \neq \hat{p}^*$. W.l.o.g. assume the merging stores are visited in a particular order, first the merging store 1, then the merging store 2 etc. all the way till the merging store k. Consider a consumer who starts searching and visits the first merging store. If the match value there is less than $\bar{x} - \hat{p}^* + \hat{p}$, the consumer will continue searching and visit a second merging firm. Otherwise, the buyer will buy right away. In the second shop, and all the way till the $(k-1)^{th}$ store the tradeoff is exactly the same. Therefore, the demand obtained by the first $k-1$ merged stores is

$$\sum_{i=1}^{k-1} \left(\bar{x}-\hat{p}^*+\hat{p}\right)^{i-1} \left(1-\bar{x}+\hat{p}^*-\hat{p}\right) = 1-\left(\bar{x}-\hat{p}^*+\hat{p}\right)^{k-1}$$

When a consumer arrives at the k^{th} merging firm, the tradeoff is different because the firm to be visited next is a non-merging firm and charges a different price, namely $\tilde{p}^*$. As a result, the consumer will search beyond the last merging firm if the highest observed utility is less than $\bar{x} - \tilde{p}^*$. In addition, we note that the other

$k-1$ shops of the merger were left because the utility levels there were less than $\bar{x}-\hat{p}^*$. Since $\hat{p}^* < \tilde{p}^*$ by assumption, this implies that some consumers may decide to return from the k^{th} merging to one of the other merging firms. This will happen when $z_{k-1}-\hat{p} > \max\{\bar{x}-\tilde{p}^*; \varepsilon_k-\hat{p}\}$. We will denote the fraction of consumers who returns to a previously visited merging store without visiting all shops in the market by $\hat{r}_m$:

$$\begin{aligned}\hat{r}_m &= (k-1)\Pr\left[\bar{x}-\hat{p}^* > z_{k-1}-\hat{p} > \max\{\bar{x}-\tilde{p}^*; \varepsilon_k-\hat{p}\}\right]\\ &= \frac{k-1}{k}\left((\bar{x}-\hat{p}^*+\hat{p})^k-(\bar{x}-\tilde{p}^*+\hat{p})^k\right)\end{aligned}$$

The sub-index m refers to the fact that these consumers return to a merging store after visiting only all the merging stores.

A consumer terminates her search in the last merging store if the match value there is higher than at the other merging stores and it is not worth to continue searching at the non-merging stores. This happens with probability

$$\begin{aligned}&\Pr\left[\varepsilon_k-\hat{p} > \max\{\bar{x}-\tilde{p}^*; z_{k-1}-\hat{p}\}\right]\\ &= (\bar{x}-\hat{p}^*+\hat{p})^{k-1}\left(1-(\bar{x}-\hat{p}^*+\hat{p})\right)+\frac{1}{k}\left((\bar{x}-\hat{p}^*+\hat{p})^k-(\bar{x}-\tilde{p}^*+\hat{p})^k\right)\end{aligned}$$

Finally, some consumers visit all shops in the market and return to one of the merging stores to conduct a purchase. This fraction of consumers, which we denote $\hat{r}_a$ to refer to the situation that consumers come back to a merging store after visiting all firms in the market, is given by:

$$\begin{aligned}\hat{r}_a &= \Pr\left[\bar{x}-\tilde{p}^* > z_k-\hat{p} > \max\{0; z_{n-k}-\tilde{p}^*\}\right]\\ &= k\int_0^{\bar{x}-\tilde{p}^*}(\varepsilon+\hat{p})^{k-1}(\varepsilon+\tilde{p}^*)^{n-k}\,d\varepsilon\end{aligned}$$

Putting the terms together, the joint payoff function of the merging stores equals

$$\hat{\pi}(\hat{p}) = \hat{p}\left[1-(\bar{x}-\tilde{p}^*+\hat{p})^k+k\int_0^{\bar{x}-\tilde{p}^*}(\varepsilon+\hat{p})^{k-1}(\varepsilon+\tilde{p}^*)^{n-k}\,d\varepsilon\right]$$

Taking the FOC and imposing the equilibrium requirement that consumer expectations are correct we obtain:

$$1-(\bar{x}-\tilde{p}^*+\hat{p}^*)^k-k(\bar{x}-\tilde{p}^*+\hat{p}^*)^{k-1}\hat{p}^*$$

$$+k\int_{0}^{\bar{x}-\tilde{p}^{*}}\left(\varepsilon+\hat{p}^{*}\right)^{k-2}\left(\varepsilon+\tilde{p}^{*}\right)^{n-k}\left(\varepsilon+k\hat{p}^{*}\right)d\varepsilon=0 \tag{2.A.25}$$

Consider now the payoff function of a non-merging firm that deviates by charging a price $\tilde{p}\neq\tilde{p}^{*}$. Since consumers expect that all non-merged firms charge the same price $\tilde{p}^{*}$, they are supposed to sample these sellers in a random way. Thus, the probability that a typical non-merging firm is visited first, second and so on till the position $n-k$ equals $1/\left(n-k\right)$. Note that conditional on arriving at a non-merging store, the probability that the buyer terminates her search there is $1-\bar{x}+\tilde{p}^{*}-\tilde{p}$. Consider a consumer who has visited all the merging stores and $h-1$ non-merging ones. The merging stores were left because the highest match value there was lower than $\bar{x}-\tilde{p}^{*}+\hat{p}^{*}$; likewise, the non-merging stores were left because the match values there were lower than $\bar{x}$. Then the demand of a merging store when it is visited in h^{th} place equals:

$$\frac{1}{n-k}\bar{x}^{h-1}\left(\bar{x}-\tilde{p}^{*}+\hat{p}^{*}\right)^{k}\left(1-\bar{x}+\tilde{p}^{*}-\tilde{p}\right)$$

Since the non-merging firm may be sampled in any position between 1 and $-k$, its direct demand equals

$$\begin{aligned}&\sum_{h=1}^{n-k}\frac{1}{n-k}\bar{x}^{h-1}\left(1-\bar{x}+\tilde{p}^{*}-\tilde{p}\right)\\&=\frac{\left(\bar{x}-\tilde{p}^{*}+\hat{p}^{*}\right)^{k}}{n-k}\frac{1-\bar{x}^{n-k}}{1-\bar{x}}\left(1-\bar{x}+\tilde{p}^{*}-\tilde{p}\right)\end{aligned}$$

A non-merging store also obtains demand from consumers who visit all sellers in the market and return to it to conduct a purchase. Denoting this demand as $\tilde{r}_a$ to refer to the fact that these consumers return to a non-merging firm after visiting all other firms in the market we have

$$\begin{aligned}\tilde{r}_a&\equiv\Pr\left[\max\left\{0;z_k-\hat{p}^{*};z_{n-k-1}-\tilde{p}^{*}\right\}<\varepsilon_i-\tilde{p}<\bar{x}-\tilde{p}^{*}\right]\\&=\int_{0}^{\bar{x}-\tilde{p}^{*}}\left(\varepsilon+\hat{p}^{*}\right)^{k}\left(\varepsilon+\tilde{p}^{*}\right)^{n-k-1}d\varepsilon\end{aligned}$$

Putting together the various sources of demand of a non-merging firm, we obtain the payoff function:

$$\tilde{\pi}=\tilde{p}\left[\frac{\left(\bar{x}-\tilde{p}^{*}+\hat{p}^{*}\right)^{k}}{n-k}\frac{1-\bar{x}^{n-k}}{1-\bar{x}}\left(1-\bar{x}+\tilde{p}^{*}-\tilde{p}\right)+\int_{0}^{\bar{x}-\tilde{p}^{*}}\left(\varepsilon+\hat{p}^{*}\right)^{k}\left(\varepsilon+\tilde{p}^{*}\right)^{n-k-1}d\varepsilon\right]$$

The corresponding FOC is:

$$\frac{(\bar{x}-\tilde{p}^{*}+\hat{p}^{*})^{k}}{n-k}\frac{1-\bar{x}^{n-k}}{1-\bar{x}}(1-\bar{x}-\tilde{p}^{*}) \tag{2.A.26}$$

$$+\int_{0}^{\bar{x}-\tilde{p}^{*}}(\varepsilon+\hat{p}^{*})^{k}(\varepsilon+\tilde{p}^{*})^{n-k-1}d\varepsilon=0 \tag{2.A.27}$$

The equilibrium prices $\hat{p}^{*}$ and $\tilde{p}^{*}$ are determined by the system of equations (2.A.25) and (2.A.27).

(a) We now prove that when search cost is sufficiently high then $\hat{p}^{*} > \tilde{p}^{*}$ and therefore consumer expectations are violated. We start by noting that, because the price of the non-merging firms is less than or equal to $1/2 < p_k^m$ for all $\bar{x} \in [p_k^m; 1]$, the integral in (2.A.25) is positive. As a result, for an equilibrium to exist, the rest of the LHS of (2.A.25),$1-(\bar{x}-\tilde{p}^{*}+\hat{p}^{*})^{k}-k(\bar{x}-\tilde{p}^{*}+\hat{p}^{*})^{k-1}\hat{p}^{*}$, must be negative. Note that this expression decreases in $\hat{p}^{*}$. Then it must be higher than when we set $\hat{p}^{*}=\tilde{p}^{*}$ because $\hat{p}^{*}<\tilde{p}^{*}$ by assumption. That is, it must be the case that

$$1-(\bar{x}-\tilde{p}^{*}+\hat{p}^{*})^{k}-k(\bar{x}-\tilde{p}^{*}+\hat{p}^{*})^{k}\hat{p}^{*}>1-\bar{x}^{k}-k\bar{x}^{k-1}\tilde{p}^{*} \tag{2.A.28}$$

We now note that when $\bar{x}\to p_k^m$ the expression $1-\bar{x}^{k}-k\bar{x}^{k-1}p_k^m$ is equal to zero. Since $\tilde{p}^{*}\geq 1/2\geq p_k^m$, it is clear that $1-\bar{x}^{k}-k\bar{x}^{k-1}p_k^m>0$ when $\bar{x}\to p_k^m$. But this constitutes a contradiction because then the LHS of (2.A.25) cannot be negative. As a result, there is no such pair of prices $\hat{p}^{*}$ and $\tilde{p}^{*}$ that satisfy (2.A.25) and (2.A.27) when $\bar{x}\to p_k^m$ and $\hat{p}^{*}<\tilde{p}^{*}$.

(b) We prove now that when search cost goes to zero again we obtain $\hat{p}^{*}>\tilde{p}^{*}$, which violates consumer expectations. To show this we use the following equality

$$Z\equiv 1-\hat{p}^{*k}\tilde{p}^{*n-k}-\hat{q}^{*}-(n-k)\tilde{q}^{*}=0$$

where $\hat{q}^{*}$ and $\tilde{q}^{*}$ denote the equilibrium quantities of the merged entity and the non-merging firms and, from the FOCs, are given by

$$\hat{q}^{*}=k(\bar{x}-\tilde{p}^{*}+\hat{p}^{*})^{k-1}\hat{p}^{*}$$
$$-k(k-1)\hat{p}^{*}\int_{0}^{\bar{x}-\tilde{p}^{*}}(\varepsilon+\hat{p})^{k-2}(\varepsilon+\tilde{p}^{*})^{n-k}d\varepsilon$$
$$\tilde{q}^{*}=\frac{(\bar{x}-\tilde{p}^{*}+\hat{p}^{*})^{k-1}}{n-k}\frac{1-\bar{x}^{n-k}}{1-\bar{x}}\tilde{p}^{*}$$

The partial derivative of Z with respect to $\hat{p}^{*}$ is negative because $\tilde{q}^{*}$ increases in

$\hat{p}^*$ and the derivative of $\hat{q}^*$ with respect to $\hat{p}^*$ is positive

$$\begin{aligned}\frac{\partial \hat{q}^*}{\partial \hat{p}^*} &= k\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1} + k\left(k-1\right)\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{k-2}\hat{p}^* \\ &- k\left(k-1\right)\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \hat{p}\right)^{k-2}\left(\varepsilon + \tilde{p}^*\right)^{n-k} d\varepsilon \\ &- k\left(k-1\right)\left(k-2\right)\hat{p}^*\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \hat{p}\right)^{k-3}\left(\varepsilon + \tilde{p}^*\right)^{n-k} d\varepsilon \\ &> k\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1} + k\left(k-1\right)\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{k-2}\hat{p}^* \\ &- k\left(k-1\right)\bar{x}^{n-k}\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \hat{p}\right)^{k-2} d\varepsilon \\ &- k\left(k-1\right)\left(k-2\right)\hat{p}^*\bar{x}^{n-k}\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \hat{p}\right)^{k-3} d\varepsilon \\ &= k\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{k-2}\left(\bar{x} - \tilde{p}^* + k\hat{p}^*\right)\left(1 - \bar{x}^{n-k}\right) > 0\end{aligned}$$

Therefore, if we set $\hat{p}^* = \tilde{p}^*$ then Z must be negative when $\bar{x} \to 1$. That is, it must be the case that

$$\begin{aligned}\lim_{\bar{x}\to 1} Z|_{\hat{p}^*=\tilde{p}^*} &= 1 - \tilde{p}^{*n} - k\tilde{p}^* + k\left(k-1\right)\tilde{p}^*\int_0^{1-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-2} d\varepsilon - \left(n-k\right)\tilde{p}^* \\ &= 1 - \tilde{p}^{*n} - n\tilde{p}^* + \frac{k\left(k-1\right)\tilde{p}^*}{n-1}\left(1 - \tilde{p}^{*n-1}\right)\end{aligned} \tag{2.A.29}$$

The FOC (2.A.27) may be rearranged as

$$\begin{aligned}&1 - \bar{x} - \tilde{p}^* + \frac{\left(n-k\right)\left(1-\bar{x}\right)}{1-\bar{x}^{n-k}}\frac{1}{\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^k} \\ &\cdot\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^k d\varepsilon = 0\end{aligned} \tag{2.A.30}$$

The LHS of (2.A.30) increases in $\hat{p}^*$ because

$$\begin{aligned}&\frac{k\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^k \int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon}{\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{2k}} \\ &- \frac{k\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1}\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^k d\varepsilon}{\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{2k}} \\ &= \frac{k\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon}{\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{k+1}}\end{aligned}$$

$$
\begin{aligned}
&- \frac{k \int_0^{\bar{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon}{(\bar{x} - \tilde{p}^* + \hat{p}^*)^{k+1}} \\
&= \frac{k \int_0^{\bar{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^{k-1} (\bar{x} - \tilde{p}^* + \hat{p}^* - \varepsilon - \hat{p}^*) \, d\varepsilon}{(\bar{x} - \tilde{p}^* + \hat{p}^*)^{k+1}} \\
&= \frac{k \int_0^{\bar{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^{k-1} (\bar{x} - \tilde{p}^* - \varepsilon) \, d\varepsilon}{(\bar{x} - \tilde{p}^* + \hat{p}^*)^{k+1}} > 0
\end{aligned}
$$

Therefore, given that $\hat{p}^* < \tilde{p}^*$, if we set $\hat{p}^* = \tilde{p}^*$ then the LHS of (2.A.30) must be positive, that is

$$
1 - \bar{x} - \tilde{p}^* + \frac{(n-k)(1-\bar{x})}{1-\bar{x}^{n-k}} \frac{1}{\bar{x}^k} \frac{1}{n} (\bar{x}^n - \tilde{p}^{*n}) > 0 \tag{2.A.31}
$$

If we take the limit of the LHS of (2.A.31) when $\bar{x} \to 1$, then we get the following inequality

$$
-\tilde{p}^* + \frac{1}{n} (1 - \tilde{p}^{*n}) > 0
$$

This inequality implies that $1 - \tilde{p}^{*n} - n\tilde{p}^* > 0$, in the limit when $\bar{x} \to 1$. This implies that (2.A.29) is positive. But this constitutes a contradiction and therefore it cannot be the case that $\hat{p}^* < \tilde{p}^*$ when $\bar{x} \to 1$.

(c) Now we prove that $\hat{p}^* > \tilde{p}^*$ if $n = 3$. We will use the results from the proof of part (b) of this proposition. If $n = 3$ then $Z(\tilde{p}^*)$ simplifies to

$$
\begin{aligned}
Z_{\hat{p}^*=\tilde{p}^*} &= 1 - \tilde{p}^{*3} - 2\bar{x}\tilde{p}^* + \tilde{p}^* \left(\bar{x}^2 - \tilde{p}^{*2}\right) - \bar{x}\tilde{p}^* \\
&= 1 - 2\tilde{p}^{*3} - 3\bar{x}\tilde{p}^* + \bar{x}^2\tilde{p}^*
\end{aligned}
$$

while condition (2.A.31) reduces to

$$
1 - \bar{x} - \tilde{p}^* + \frac{1}{3\bar{x}^2} \left(\bar{x}^3 - \tilde{p}^{*3}\right) > 0
$$

or

$$
\tilde{p}^* < 1 - \bar{x} + \frac{1}{3\bar{x}^2} \left(\bar{x}^3 - \tilde{p}^{*3}\right) < 1 - \bar{x} + \frac{1}{3\bar{x}^2} \left(\bar{x}^3 - (1-\bar{x})^3\right)
$$

Then

$$
Z_{\hat{p}^*=\tilde{p}^*} > 1 - 2\tilde{p}^{*3} + \left(\bar{x}^2 - 3\bar{x}\right) \left(1 - \bar{x} + \frac{\bar{x}^3 - \tilde{p}^{*3}}{3\bar{x}^2}\right)
$$

$$= 1 + \tilde{p}^{*3} \frac{3 - 7\bar{x}}{3\bar{x}} - \frac{\bar{x}}{3} (3 - \bar{x}) (3 - 2\bar{x})$$
$$> 1 + \left(\frac{1}{2}\right)^3 \frac{3 - 7\bar{x}}{3\bar{x}} - \frac{\bar{x}}{3} (3 - \bar{x}) (3 - 2\bar{x})$$
$$= \frac{1}{24\bar{x}} \left(3 + 17\bar{x} - 72\bar{x}^2 + 72\bar{x}^3 - 16\bar{x}^4\right) \tag{2.A.32}$$

which is always positive as shown in Figure 2.A.4

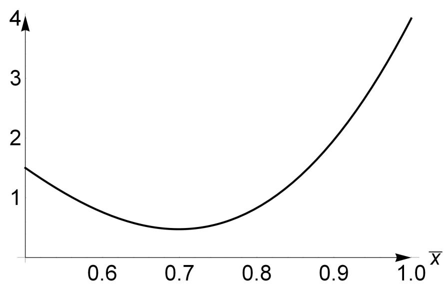


Figure 2.A.4. Plot of expression (2.A.32).

(d) Finally, in the limit when $n \to \infty$ the FOC of the merged entity becomes

$$1 - (\bar{x} - \tilde{p}^* + \hat{p}^*)^k - k (\bar{x} - \tilde{p}^* + \hat{p}^*)^{k-1} \hat{p}^* = 0 \tag{2.A.33}$$

while that of a non-merged firm becomes

$$(\bar{x} - \hat{p}^* + \tilde{p}^*)^k (1 - \bar{x} - \tilde{p}^*) = 0$$

This implies that $\lim_{n\to\infty} \tilde{p}^* = 1 - \bar{x}$.

The LHS of (2.A.33) decreases in $\hat{p}^*$. Then, if $\hat{p}^* < \tilde{p}^*$ then the LHS of (2.A.33) must be negative if we replace $\hat{p}^*$ by $\tilde{p}^* = 1 - \bar{x}$. However,

$$1 - \bar{x}^k - k\bar{x}^{k-1} (1 - \bar{x}) = 1 + (k - 1) \bar{x}^k - k\bar{x}^{k-1} \geq 0$$

where the inequality follows from setting $\bar{x} = 1$. This establishes a contradiction so $\hat{p}^* < \tilde{p}^*$ cannot hold in the limit when $n \to \infty$. ■.

Chapter 3

Horizontal Mergers and Economies of Search*

3.1 Introduction

Often firms that merge, after a more or less complex process of business reorganization, choose to shut down shops and crowd products together. Even though this process may a priori be driven by a desire to achieve cost savings, we put forward a different, possibly complementary, rationale: when consumer search costs are significant, crowding products together generates *demand-side economies.* This paper studies, on the one hand, how firms can benefit from the consumer search economies generated by horizontal mergers and, on the other hand, the aggregate implications of merging activity.

We study a consumer search market where a few firms sell differentiated products. Firms compete in prices and consumers search for satisfactory deals sequentially. In the pre-merger symmetric market, all firms look alike and when consumers pick a first shop to visit, they do it in a random way. Those consumers who fail to find a satisfactory deal continue searching and once again they pick the next shop to visit randomly; and so on. This model was introduced by Wolinsky (1986) and was further studied by Anderson and Renault (1999).

When firms merge and crowd products together the following trade-off for a consumer arises: relative to the deal offered by a non-merging firm, at the merged entity, the consumer encounters more variety though likely offered at a higher

*This chapter is based on Moraga-González and Petrikaitė (2011b) price. We show that this trade-off ends up being favorable for the merging firms when search costs are relatively high. In the unique equilibrium of the post-merger market consumers find it optimal to search first at the merged entity and then, if unsatisfied with the deals available there, continue searching among the non-merging stores.

Search costs, even if small, are known to have important implications for the functioning of markets. In our model, merging is individually rational and, in contrast to most papers on mergers, the outsiders' profits decrease after a merger takes place. Moreover, consumers may benefit from consolidation in the market: we show that the economies from lower search costs may outweigh the price-rising effects of the merger.

The literature on the incentives to merge and the aggregate implications of mergers is quite extensive. For a recent survey of the main theoretical and empirical insights see Whinston (2006). A seminal paper in the literature is Salant et al. (1983), who demonstrated that merging is not very attractive in environments where firms compete in quantities and offer similar products. This result is so surprising that is referred to as the *merger paradox*. Deneckere and Davidson (1985) showed that price-setting firms selling horizontally differentiated products, other things equal, always have an incentive to merge. In contrast to the Cournot case analyzed by Salant et al. (1983), this result arises because price increases of the merging firms, which favor the coalition partners, are accompanied by price increases of the non-merging firms, which also favors them.

In our model firms also compete in prices and sell differentiated products. However, sufficiently high search costs give a rise to interesting results. When search costs are significant. In that case, by clustering all products together, mergers serve to effectively lower the costs of searching the products of the potentially merging firms. This, everything else equal, gives the merged entity a *prominent* position in the market, which implies that the merged entity attracts all consumer first-visits. In some cases, despite having to internalize the pricing externalities among all its goods, we show that the prominence effect may be so strong that the merged entity may charge lower prices than the non-merging firms (as it actually occurs in Armstrong et al. (2009) paper on the effects of prominence). In contrast to Deneckere and Davidson (1985) analysis, in our paper insiders obtain larger gains than outsiders if a merger occurs. This is because consumers postpone visiting the outsiders until they have visited the merged entity. When search costs are high, consumer traffic from the merged entity to the non-merging firms is so small that the outsiders lose out. On the consumer welfare side, mergers have the potential to generate sufficiently large search economies so as to benefit consumers too.

Since the seminal paper of Williamson (1968), the role of mergers at generating supply-side economies, or cost-synergies, that can more than offset the market power effects of consolidation has been the focus of a considerable amount of research. Perry and Porter (1985), Farrel and Shapiro (1990) and McAfee and Williams (1992) explicitly modeled the cost efficiencies that arise from economies of sharing assets in product markets and stated conditions for an *efficiency defense* of mergers. Our analysis also brings out an efficiency defence argument of mergers, but based on demand- rather than on supply-side economies.

Section 5.7 of the 2010 Merger Assessment Guidelines of the U.K. Competition Commission and the Office of Fair Trading acknowledges the importance of demand-side efficiencies in merger control. However, the guidelines focus mainly on cases where complementarities are significant: "*Demand-side efficiencies arise if the attractiveness to customers of the merged firm's products increases as a result of the merger. Common examples of demand-side efficiencies include: network effects, pricing effects and 'one-stop shopping'.*[1] In this chapter we show that demand-side economies can also arise even if products are substitutes. The reason is that shops that carry more variety can, in spite of the potential negative price effects, be more attractive for consumers.

This chapter is related to a strand of the consumer search literature dealing with firm's choice of location, entry and choice of product-lines in the presence of search costs. This chapter and those papers have in common that consumer search economies play a central role. In Stahl (1982) and Wolinsky (1983) savings in search costs can explain the observed geographical concentration of stores selling differentiated products. Fischer and Harrington Jr (1996) go one step further and investigate the role of product heterogeneity in explaining interindustry variation in firm agglomeration. Dudey (1990) studies the case of homogeneous products and finds conditions under which firms cluster at a single location. Wilson (2010), by contrast, shows that homogeneous product firms may have an incentive to "obfuscate" the market by locating in less-easy-to-reach locations. Like in Fischer and Harrington Jr (1996), Non (2010) reconciles these two ideas by showing that clustered and peripheral homogeneous product firms can coexist in the market. Ellison

[1] With *network effects*, users place the higher value for a product the more it is used by other consumers. A merger may make networks compatible and so enhance the welfare of consumers. *Pricing effects* arise when bringing complement products under common ownership, which may results in lower prices for all products. Gains from *one-stop shopping* arise when consumers have a strong preference for buying a range of products at a single supplier.

and Wolitzky (2009) also study obfuscation strategies in the market. They argue that consumer search dis-economies can very well explain the obfuscation strategies observed in Ellison and Ellison (2009). Economies of scope in search costs also play a central role in explaining entry patters and the choice of product-lines. Schultz and Stahl (1996) show that economies of scope in search costs can lead to excessive (price-increasing) entry. Economies of scope in shopping costs also arise when consumers buy many products and prefer to concentrate their purchases with a single supplier. Klemperer (1992) shows that in these situations firms may prefer head-to-head competition over product-line differentiation. In a subsequent paper, Klemperer and Padilla (1997) show that search cost economies can lead to excessive product-line variety.

This chapter is also related to a recent literature on ordered search. Arbatskaya (2007) studies a market for homogeneous products where the order in which firms are visited is exogenously given. In equilibrium prices must fall as the consumer walks away from the firms visited first. Zhou (2011) considers the case of differentiated products and finds the opposite result. As mentioned above, Armstrong et al. (2009) study the implications of "prominence" in search markets. In their model, there is a firm that is always visited first and this firm charges lower prices and derives greater profits than the rest of the firms, which are visited randomly after consumers have visited the prominent firm. Zhou (2009) extends the ideas in Armstrong et al. (2009) to the case in which a set of firms, rather than just one, is prominent. His analysis shares some features with our model because the merging firms, by crowding products together, become "prominent". Haan and Moraga-González (2011) study a model where firms compete in advertising to raise consumer attention. The firms whose advertising is more salient gain market prominence. Consumers visit them earlier along the search process and charge lower prices. They show that firms need not benefit from higher search costs.

The remainder of the chapter is organized as follows. Section 3.2 describes the consumer search model. Section 3.3 presents the benchmark equilibrium of the pre-merger market. Section 3.4 discusses our main results. 3.5 briefly concludes the chapter. To ease the reading of the chapter, long proofs are placed in an appendix.

3.2 The model

We study the search model for differentiated products first proposed by Wolinsky (1986) and further studied by Anderson and Renault (1999).[2] On the supply side of the market there are n firms selling horizontally differentiated products. All firms employ the same constant returns to scale technology of production and we normalize unit production costs to zero. On the demand side of the market, there is a unit mass of consumers. A consumer m has tastes described by an indirect utility function

$$u_{mi}(p_i) = \varepsilon_{mi} - p_i,$$

if she buys product i at price p_i. The parameter ε_{mi} can be thought of as a match value between consumer m and product i. We assume that the value ε_{mi} is the realization of a random variable uniformly distributed on $[0, 1]$. Match values are independently distributed across consumers and products. No firm can observe ε_{mi} so personalised pricing is not possible. In what follows we will denote $z_k \equiv \max\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_k\}$. For later reference, it is useful to calculate the optimal price of a multi-product monopolist selling k varieties, which we denote p_k^m. This price maximizes $p(\Pr[z_k \geq p])$, which gives $p_k^m = (1 + k)^{-\frac{1}{k}}$. Setting $k = 1$, we have the single-product monopolist, whose price is simply denoted by p^m and is equal to $1/2$.

Consumers search sequentially with costless recall. Each time a consumer searches, she must pay a search cost denoted s. To avoid that a market equilibrium fails to exist (Diamond, 1971), we assume that search cost s is relatively small. In particular:

Assumption SC. *We assume that $s \in [0, 1/8]$; moreover, we assume that the number of firms $n \leq 10$.*[3]

This assumption ensures that the first search is always worthwhile for any number of varieties, k, sold by the first-visited shop, that is:

$$\Pr[z_k \geq p_k^m] E[z_k - p_k^m \mid z_k \geq p_k^m] - s$$

[2] More recently, this model has been used to explain incentives to invest in quality (Wolinsky, 2005); product-designs (Bar-Isaac, Caruana, and Cuñat, 2011) and the emergence and the effects of market prominence (Armstrong et al., 2009; Haan and Moraga-González, 2011; Zhou, 2009; Zhou, 2011).

[3] The restriction $n \leq 10$ is mainly technical and serves to avoid situations in which a post-merger equilibrium fails to exist. The reason is as follows. Fix the search cost s and suppose n is sufficiently large. Then an arbitrarily large k-firm merger has an incentive to charge such a high price that the buyer will find it unprofitable to enter the market. This produces a Diamond-paradox type of result and the equilibrium collapses. In practice, and for the purpose of this paper, since mergers are relevant in relatively concentrated markets, the restriction $n \leq 10$ implies little loss of generality.

$$= \int_{p_k^m}^{1} (z_k - p_k^m) k\varepsilon^{k-1} d\varepsilon - s \geq 0 \text{ for all } k = 1, 2, ..., n$$

For $k = 1$, this inequality holds strictly if $s = 1/8$. For larger k values, the expected utility a consumer derives from her first search always covers the search cost.

3.3 Pre-merger market

As a benchmark case, in this section we characterize the pre-merger market symmetric equilibrium.

Let $p^* \in [0, p^m]$ denote the prices of firms other than firm i. Consider the (expected) payoff to a firm i that deviates by charging a price p_i. Assume $p_i \geq p^*$ without loss of generality. We start by computing the probability that a consumer accepts the offer of firm i, conditional on visiting firm i first. For this, we need to characterize the optimal consumer stopping rule. Suppose that a buyer has arrived at a certain firm and her current most favorable purchase option gives her utility $\varepsilon_i - p_i$. If $\varepsilon_i - p_i < 0$, the consumer will search again given our assumption $s < 1/8$. Suppose $\varepsilon_i - p_i \geq 0$. In the Nash equilibrium, a visit to a new firm will yield utility $\varepsilon - p^*$. Kohn and Shavell (1974) show that the consumer should continue to search if her best previously discovered match value ε_i is lower than $\overline{x}$, where $\overline{x}$ is the solution to the equation

$$\int_{x}^{1} (\varepsilon - x) d\varepsilon = s, \tag{3.1}$$

that is, $\overline{x} = 1 - \sqrt{2s}$. To see this, notice that searching one more time yields gains only if the consumer prefers the new option, say j, over option i, i.e., if $\varepsilon_j > \varepsilon_i - p_i + p^*$. Denoting $x \equiv \varepsilon_i - p_i + p^*$, the expected benefit from searching once more is $\int_x^1 (\varepsilon - x) d\varepsilon$, which is the LHS of (3.1). Searching one more time is worthwhile if and only if these incremental benefits exceed the cost of search s. Therefore, the buyer is exactly indifferent between searching once more and stopping and accepting the offer at hand if $x = \overline{x}$. Since $s \in [0, 1/8]$, we have that $\overline{x} \in [1/2, 1]$.

In any symmetric equilibrium, it must be the case that $\overline{x} \geq p^*$ for otherwise no consumer would participate in the market. Given this, the probability that a buyer stops searching at firm i given that firm i is visited in first place is equal to

$$\Pr[x > \overline{x}] = 1 - (\overline{x} + p_i - p^*), \tag{3.2}$$

provided the deviating price is not too high, i.e., $p_i < 1 - \overline{x} + p^*$, for otherwise every single consumer would walk away from firm i.[4]

Before visiting firm i, the consumer may have visited other firm(s). The probability that a consumer goes to firm i in her second search and decides to acquire the offering of firm i right away is $\overline{x}(1-\overline{x}-p_i+p^*)$.[5] Similarly, the probability that a consumer goes to firm i in her h-th search and decides to acquire the offering of firm i right away is $\overline{x}^{h-1}(1-\overline{x}-p_i+p^*)$.

To complete firm i's payoff calculation, we need to compute the joint probability that a consumer walks away from every single firm in the market and happens to return to firm i to conduct a transaction, that is

$$\Pr[\max\{0, z_{n-1}-p^*\} < \varepsilon_i - p_i < \overline{x}-p^*] \tag{3.3}$$

This probability is independent of the order in which firms are visited so we will denote it as $R(p_i;p^*)$. We then have:

$$R(p_i;p^*) = \int_{p_i}^{\overline{x}+p_i-p^*} (\varepsilon_i - p_i + p^*)^{n-1} d\varepsilon_i = \int_0^{\overline{x}-p^*} (\varepsilon_i + p^*)^{n-1} d\varepsilon_i = \frac{1}{n}(\overline{x}^n - p^{*n}).$$

We can now write the deviant firm's expected profits:

$$\Pi_i(p_i;p^*) = \frac{p_i}{n}\left[\frac{1-\overline{x}^n}{1-\overline{x}}(1-\overline{x}-p_i+p^*) + (\overline{x}^n - p^{*n})\right].$$

We look for a symmetric Nash equilibrium in prices. Hence, the first-order condition is:

$$1 - p^{*n} - p^*\frac{1-\overline{x}^n}{1-\overline{x}} = 0 \tag{3.4}$$

It is easy to check that (3.4) has a unique solution that satisfies $\overline{x} \geq p^* \geq 1-\overline{x} \geq 0$.Since the LHS of (3.4) decreases in $\overline{x}$, the equilibrium price increases in the search cost s. In the limit when $s = 1/8, \overline{x} = p^* = 1/2$.

The profits of a typical firm in the pre-merger situation are

$$\pi^* = \frac{1}{n}p^*(1-p^{*n}). \tag{3.5}$$

[4] In what follows we derive the payoff of a firm under the assumption that $p_i < 1-\overline{x}+p^*$. When this does not hold, the payoff is slightly different. See footnote 4).

[5] Letting j denote the firm visited earlier, this probability is given by $\Pr[\varepsilon_i - p_i > \overline{x} - p^* > \varepsilon_j - p^*]$.

3.4 Mergers of k firms

In this section we study the price implications of mergers and the incentives to merge. Consider that k out of the n active firms merge, with $2 \leq k \leq n-1$. We take a long-term view of mergers and assume that the k merging stores shut down all their shops but one, where they crowd the k varieties stemming from the k original merging firms together. In what follows, a typical non-merging store will be labeled j.

Let $\tilde{p}^* \in [0, p^m]$ and $\hat{p}^* \in [0, p_k^m]$ denote the equilibrium prices charged by the non-merging firms and the merged entity, respectively. To characterize the post-merger equilibrium we need to write out the payoffs of the different types of firms. These payoffs in turn depend on the optimal consumer search behavior, which, of course, has to be consistent with equilibrium pricing.

We then proceed by first specifying the order in which consumers will visit the various types of firms, then calculating equilibrium prices and finally checking back the consistency of the search rule. The trade-off for a consumer is clear: relative to the deal offered by a non-merging firm, at the merged entity, the consumer encounters more variety but possibly at higher prices,[6] offered at a higher price.

Let $\overline{\overline{x}}$ be the solution to

$$\int_x^1 (\varepsilon - x) d\varepsilon^k - s = 0 \tag{3.6}$$

As in (3.1), $\overline{\overline{x}}$ represents a threshold value above which a consumer will decide not to continue searching the products of the merged entity.

Define the $\overline{x} - \tilde{p}^*$ as the reservation utility for searching a non-merging store. Note that in any equilibrium where the non-merging stores have positive market shares, it must be the case that $\overline{x} - \tilde{p}^* \geq 0$. Likewise, the number $\overline{\overline{x}} - \hat{p}^*$ is the corresponding reservation utility at the merged entity. Weitzman (1979)'s paper on optimal search for the best alternative prescribes the consumer should search as follows: at every step in the search process a consumer should consider visiting next the (not-yet-visited) shop for which reservation utility is the highest; moreover,

[6] Merging firms internalize pricing externalities so it is reasonable to expect lower prices at the non-merging stores. However, there is a recent literature on oligopolistic competition with search frictions showing that firms that are visited first charge lower prices than the rivals that are visited later (Armstrong et al., 2009; Haan and Moraga-González, 2011; Zhou, 2009). Therefore, if the merged entity were to be visited first by the consumers in an equilibrium, it may very well be the case that it ends up charging a lower price than the non-merging stores.

at every step in the search process a consumer should terminate search whenever the maximum sampled reward so far is above the reservation utility at the shop to be visited next.

Momentarily, assume $\overline{\overline{x}} - \hat{p}^* > \overline{x} - \tilde{p}^*$ so that consumers visit first the merged entity. To calculate the post-merger equilibrium prices, we proceed by computing the payoffs of the firms (merging and non-merging) which they would obtain when deviating from the equilibrium prices. Then we derive the first order conditions (FOCs), require consumer expectations to be correct, and solve for equilibrium prices. After this we look for conditions under which $\overline{\overline{x}} - \hat{p}^* > \overline{x} - \tilde{p}^*$ indeed holds. Later in Section 3.4.2 we prove that the equilibrium we derive here is unique provided that search costs are sufficiently large.

Payoff to a deviant merging store

Since consumers expect the price set by the merged entity to be $\hat{p}^*$, given our assumption SC, they will make the first search. Upon arrival at the merged entity, they may be surprised by a deviation, denoted $\hat{p}$. Without loss of generality, suppose that the deviating price $\hat{p} < \hat{p}^*$.

Let $z_k - \hat{p}$ be the deal observed by the consumer at the merged entity. A consumer stops there and buys right away if the expected gains from further search are lower than the search cost. Applying the same logic as in equation (3.2), since consumers believe the non-merging firms to be charging $\tilde{p}^*$, the probability that a buyer stops searching at the merged entity is equal to

$$1 - (\overline{x} - \tilde{p}^* + \hat{p})^k, \tag{3.7}$$

provided, again, that the deviating price is not too high.

The merged entity also receives demand from consumers who decide to walk away from it and venture the non-merging firms only to find out that the deal offered by the merged entity is in the end the best in the market. This happens with probability:

$$\Pr\left[z_k - \hat{p} < \overline{x} - \tilde{p}^* \text{ and } z_{n-k} < \overline{x} \text{ and } z_k - \hat{p} > \max\{z_{n-k} - \tilde{p}^*, 0\}\right],$$

which is equivalent to

$$\Pr\left[z_k - \hat{p} < \overline{x} - \tilde{p}^* \text{ and } z_k - \hat{p} > \max\{z_{n-k} - \tilde{p}^*, 0\}\right]$$

$$= \int_{\hat{p}}^{\overline{x}-\tilde{p}^*+\hat{p}} (\varepsilon - \hat{p} + \tilde{p}^*)^{n-k} \, d\varepsilon^k = k \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p})^{k-1} \, d\varepsilon \tag{3.8}$$

The demand of the merged entity is therefore the sum of (3.7) and (3.8). Therefore, the total profit of the merged entity equals:

$$\hat{\pi}(\hat{p}) = \hat{p} \left[1 - (\overline{x} - \tilde{p}^* + \hat{p})^k + k \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p})^{k-1} \, d\varepsilon \right] \tag{3.9}$$

Payoff to a deviant non-merging store

We now compute the payoff of a non-merging store that deviates from $\tilde{p}^*$ by charging $\tilde{p}$. Without loss of generality assume $\tilde{p} < \tilde{p}^*$. Once consumers walk away from the merged entity, as all non-merging firms are supposed to charge the same price $\tilde{p}^*$, consumers are assumed to visit them randomly. Therefore the deviant firm may be visited in the first place after the merged entity, in the second place and so on till the $n - k^{th}$ place. Like any other non-merging store, the deviant has a probability $1/(n-k)$ of being visited in each of these places.

Consider that the deviant non-merging firm is visited by a consumer in her h-th search after walking away from the merged entity, with $h = 1, 2, ..., n-k$.[7] Suppose the deal the consumer observes upon entering the deviant's shop is $\varepsilon_j - \tilde{p}$. There are two situations in which the deviant sells to this consumer. First, the consumer may stop searching at this shop and buy there right away. Using the search logic described above, conditional on the consumer visiting non-merging firm j in her h-th search, this occurs when $\varepsilon_j \geq \overline{x} - \tilde{p}^* + \tilde{p}$. Therefore, the joint probability a consumer visits the deviant in h-th place and buys there directly is

$$\begin{aligned} &\frac{1}{n-k} \Pr[z_k - \hat{p}^* < \overline{x} - \tilde{p}^* \text{ and } z_{h-1} < \overline{x} \text{ and } \varepsilon_j - \tilde{p} > \overline{x} - \tilde{p}^*] \\ &\qquad = \frac{1}{n-k} (\overline{x} - \tilde{p}^* + \hat{p}^*)^k \overline{x}^{h-1} (1 - \overline{x} + \tilde{p}^* - \tilde{p}) \end{aligned}$$

Second, the consumer may walk away from the deviant firm and come back to it

[7] We note that when the consumer visits the deviant immediately after leaving the merged firm, $h = 1$, the consumer, even if surprised by a deviation, never wants to return to the merged entity without searching further. If fact, this event has probability

$$\Pr[z_k - \widehat{p}^* < \overline{x} - \widetilde{p}^* \text{and } \varepsilon - \widetilde{p} > \overline{x} - \widetilde{p}^* \text{ and } z_k - \widehat{p}^* > \varepsilon - \widetilde{p}] = 0$$

after visiting all non-merging stores. This occurs when

$$\Pr\left[z_k - \hat{p}^* < \overline{x} - \tilde{p}^* \text{ and } z_{n-k-1} < \overline{x} \text{ and } \varepsilon_j - \tilde{p} < \overline{x} - \tilde{p}^* \right.$$
$$\left. \text{and } \varepsilon_j - \tilde{p} > \max\left\{z_k - \hat{p}^*, z_{n-k-1} - \tilde{p}^*, 0\right\}\right]$$

which after manipulation gives

$$\begin{aligned}
&\Pr\left[\overline{x} - \tilde{p}^* + \tilde{p} > \varepsilon_j > \max\{z_k - \hat{p}^* + \tilde{p}, z_{n-k-1} - \tilde{p}^* + \tilde{p}, \tilde{p}\}\right] \\
&= \int_{\tilde{p}}^{\overline{x}-\tilde{p}^*+\tilde{p}} (\varepsilon_j + \hat{p}^* - \tilde{p})^k (\varepsilon_j + \tilde{p}^* - \tilde{p})^{n-k-1} d\varepsilon_j \\
&= \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon_j + \hat{p}^*)^k (\varepsilon_j + \tilde{p}^*)^{n-k-1} d\varepsilon_j
\end{aligned}$$

Note that this probability does not depend on h.

Taking into account that, once the consumer has walked away from the merged entity, the deviant may be visited by her in positions $h = 1, 2, ..., n-k$ we have a total demand for the non-merging firm equal to

$$\frac{1}{n-k}\sum_{h=1}^{n-k}(\overline{x} - \tilde{p}^* + \hat{p}^*)^k \overline{x}^{h-1}\left(1 - \overline{x} + \tilde{p}^* - \tilde{p}\right) + \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \hat{p}^*)^k (\varepsilon + \tilde{p}^*)^{n-k-1} d\varepsilon$$

Using this expression, we obtain the profits of the non-merging firm:

$$\begin{aligned}
\tilde{\pi}(\tilde{p}) \quad &= \quad \tilde{p}\left(\frac{1}{n-k}\frac{1-\overline{x}^{n-k}}{1-\overline{x}}(\overline{x} - \tilde{p}^* + \hat{p}^*)^k\left(1 - \overline{x} + \tilde{p}^* - \tilde{p}\right)\right. \\
&+ \quad \left.\int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \hat{p}^*)^k (\varepsilon + \tilde{p}^*)^{n-k-1} d\varepsilon\right) \qquad (3.10)
\end{aligned}$$

Taking the FOCs in (3.9) and (3.10) and requiring that consumer expectations are fulfilled, i.e. $\hat{p} = \hat{p}^*$ and $\tilde{p} = \tilde{p}^*$, gives:

$$\begin{aligned}
&1 - \left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1}\left(\overline{x} - \tilde{p}^* + (k+1)\,\hat{p}^*\right) \\
+ \quad & k\int_0^{\overline{x}-\tilde{p}^*} \left(\varepsilon + \tilde{p}^*\right)^{n-k}\left(\varepsilon + \hat{p}^*\right)^{k-2}\left(\varepsilon + k\hat{p}^*\right) d\varepsilon = 0 \qquad (3.11) \\
&\frac{1}{n-k}\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^k \frac{1-\overline{x}^{n-k}}{1-\overline{x}}\left(1 - \overline{x} - \tilde{p}^*\right) \\
+ \quad & \int_0^{\overline{x}-\tilde{p}^*} \left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^k d\varepsilon = 0 \qquad (3.12)
\end{aligned}$$

3.4.1 Main results: high search costs

The main results of this article pertain to the case where search costs are significant. In this section we focus on such situations.

Proposition 3.1. *Assume that $k \leq n-1$ firms merge. Then there exists a Nash equilibrium in the post-merger market where:*

- *Consumers prefer to search first the products of the merged entity and then, if they wish so, they proceed by searching the products of the non-merging firms.*
- *The merged entity charges a price $\hat{p}^*$ and the non-merging stores charge a price $\widetilde{p}^*$; these prices solve the system of first order conditions (3.11)-(3.12).*

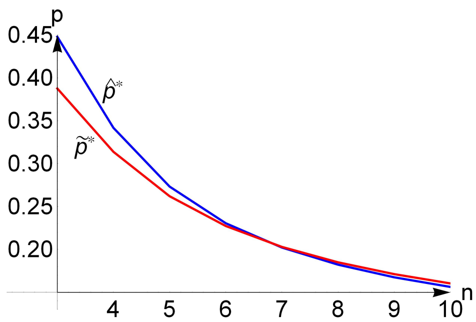
This equilibrium exists if the search cost s is sufficiently large, in which case $\hat{p}^* \geq \widetilde{p}^*$.

The proof, which is presented in the appendix, has three steps. We first prove that there exists at least one solution to the system of first order conditions (3.11)-(3.12). We then show that this solution is unique. Finally, we show that when the search cost is large, consumer putative search order (first the merged entity then the non-merging firms) is consistent with equilibrium pricing.

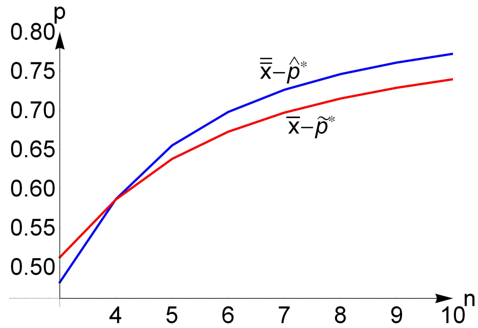
Before turning to a discussion of the aggregate implications of mergers, we make two remarks in connection with Proposition 3.1. The first observation is that, even though the proof of the proposition uses the case where the search cost is very high and converges to its maximum value, the result is true for much lower search costs. The second observation is that the ranking of merging and non-merging firm prices can be different than the one in Proposition 3.1. It is indeed possible that the search-order effect more than offsets the internalization-of-pricing-externalities effect, in which case the merged entity charges a price lower than the non-merging stores. This occurs when the search cost is relatively small and the number of merging firms relative to the total number of firms in the market is also small. These two remarks can be seen in the graphs of Figure 3.1. In these two graphs, the number of merging firms is set to 2 and the search cost is very small ($s = 0.005$). Figure 3.1a plots the post-merger equilibrium prices and shows that the merged entity charges a price lower than that of the non-merging when n is relatively large. Figure 3.1b plots consumer reservation utilities for searching the two types of firm, which shows that consumer search order is consistent with equilibrium pricing for all $n \geq 4$.

We now turn to study the incentives to merge.

Proposition 3.2. *For the Nash equilibrium of Proposition 3.1 we have:*



(a) Prices



(b) Reservation utilities

Figure 3.1. Post-merger prices and reservation utilities ($\overline{x} = 0.9, k = 2$)

- *Any k-firm merger is individually rational for the merging firms, that is,* $\hat{\pi}^*/k > \pi^*$.
- *If the search cost is sufficiently large, in any k-firm merger the non-merging firms obtain lower profits than the merging firms, that is,* $\hat{\pi}^*/k > \tilde{\pi}^*$.

The proof is in the appendix. As shown by Deneckere and Davidson (1985), in models of price competition with differentiated products merging is individually rational. Our first observation in Proposition 3.2 generalizes this insight to the case where there exist search frictions in the market. Our second result in Proposition 3.2 is that non-merging firms may obtain lower profits than the merging ones. This observation contrasts earlier work and deserves an explanation. Here, by merging, the potentially merging firms gain prominence in the market so that in the equilibrium of Proposition 3.1 the merged entity is visited first. This is to the detriment of the non-merging firms, which are relegated to the end of the optimal search order that consumers follow when they search for satisfactory deals. When search costs are significant, the search-order effect is substantial: the non-merging firms receive too few visitors and lose out relative to the merging firms.

Our final result pertains to the aggregate implications of mergers. As usual, we evaluate the effects of a merger on welfare grounds by comparing the pre- and post-merger un-weighted sum of consumer surplus and firms' profits. We now compute the expected surplus consumers derive in the post-merger market. Consider first those consumers who end up buying from the merged entity. As explained above, these consumers either buy directly, in which case they incur the search cost only one time, or they buy at the merged entity after having visited all the non-merging firms, in which case they incur a total search cost of $(n-k+1)\,s$. Denoting by $\widehat{CS}$

the expected surplus of the clientele of the merged entity we have:

$$\begin{aligned}\widehat{CS} &= \int_{\overline{x}-\tilde{p}^*+\hat{p}^*}^{1} (\varepsilon - \hat{p}^* - s)\, d\varepsilon^k \\ &+ \int_{\hat{p}^*}^{\overline{x}-\tilde{p}^*+\hat{p}^*} (\varepsilon - \hat{p}^* + \tilde{p}^*)^{n-k} (\varepsilon - \hat{p}^* - (n-k+1)\, s)\, d\varepsilon^k \end{aligned} \tag{3.13}$$

Consider now those consumers who end up buying from the non-merging firms. These consumers may buy directly at one of the non-merging firms after walking away from the merged entity, in which case they incur a total search cost of $2s$; or else they may leave some ℓ non-merging firms to finally buy at the $\ell+1$-th one, in which case the total search cost they incur is equal to $(\ell+2)s$; or finally, they may walk away from all merging and non-merging firms only to find out that the best deal is at one of the non-merging firms, in which case they incur a total search cost of $(n-k+1)\, s$. Denoting by $\widetilde{CS}$ the expected consumer surplus of the clientele of the non-merging firms we obtain

$$\begin{aligned}\widetilde{CS} &= (\overline{x} - \tilde{p}^* + \hat{p}^*)^k \sum_{\ell=1}^{n-k} \overline{x}^{\ell-1} \int_{\overline{x}}^{1} (\varepsilon - \tilde{p}^* - (\ell+1)s) d\varepsilon \\ &+ (n-k) \int_{\tilde{p}^*}^{\overline{x}} \varepsilon^{n-k-1} (\varepsilon - \tilde{p}^* + \hat{p}^*)^k (\varepsilon - \tilde{p}^* - (n-k+1)\, s)\, d\varepsilon \\ &= (\overline{x} - \tilde{p}^* + \hat{p}^*)^k \frac{1-\overline{x}^{n-k}}{1-\overline{x}} \int_{\overline{x}}^{1} (\varepsilon - \tilde{p}^*)\, d\varepsilon \\ &- (\overline{x} - \tilde{p}^* + \hat{p}^*)^k (1-\overline{x})\, s \sum_{\ell=1}^{n-k} (\ell+1) \overline{x}^{\ell-1} \\ &+ (n-k) \int_{\tilde{p}^*}^{\overline{x}} \varepsilon^{n-k-1} (\varepsilon - \tilde{p}^* + \hat{p}^*)^k (\varepsilon - \tilde{p}^* - (n-k+1)\, s)\, d\varepsilon. \end{aligned} \tag{3.14}$$

Some consumers do not buy at all. The mass of these consumers is $\hat{p}^{*k}\tilde{p}^{*n-k}$. By construction, they visit all the firms in the market. Hence, their consumer surplus is negative and equals

$$CS_{\varnothing} = -\hat{p}^{*k}\tilde{p}^{*n-k}\, (n-k+1)\, s \tag{3.15}$$

Aggregating the surplus over the different consumers and firms, we obtain a measure of expected social welfare

$$SW = \widehat{CS} + \widetilde{CS} + CS_{\varnothing} + \hat{\pi} + (n-k)\tilde{\pi}.$$

Proposition 3.3. *For the Nash equilibrium of Proposition 3.1, if the search cost is sufficiently high we have that:*

- *Any k-firm merger results in an increase in industry profits.*
- *Consumer surplus increases after a k-firm merger.*

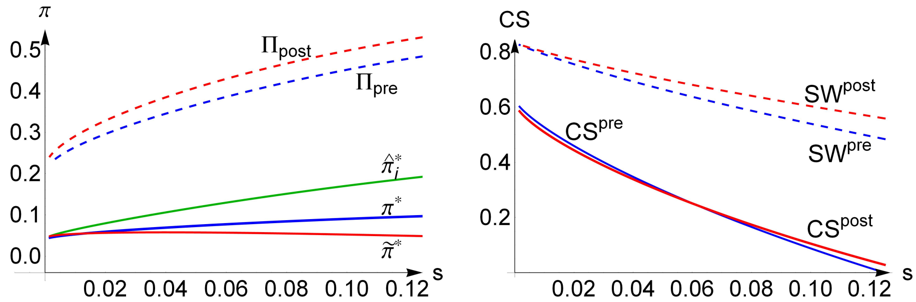
As a result, social welfare increases after a merger has taken place.

The proof is in the appendix. The aggregate implications of a merger are illustrated in Figure 3.2. The first observation is that the results in Proposition 3.3 not only hold for very high search costs but more generally. In Figure 3.2a we compare pre- and post-merger (individual and collective) profits. It can be seen that the merged entity's profits (green curve) are clearly above pre-merger levels (blue solid curve). As explained before, this is the outcome of two forces: one the one hand, the merged entity benefits from the market prominence it gains by clustering products together; on the other hand, the merged entity profits from increased market power. The figure also shows that when search costs are not extremely low, outsiders lose out (red solid curve). In any case, collectively firms obtain greater profits post-merger (red dashed curve) than pre-merger (blue dashed curve). Finally, it is also worth mentioning that the asymmetry in the way search costs affect the profits of the different firms after a merger. As search costs increase, the profits of the merged entity go up while the profits of the non-merging firms typically fall down. This is due to the fact that as search costs increase consumer traffic from the merged entity to the non-merging firms falls.

Figure 3.2b depicts pre- and post-merger consumer surplus and social welfare. The graph illustrates our result in Proposition 3.3 that when a search cost is high, consumer search economies more than offset the negative price effects of consolidation. When search costs are low, consumers lose but overall welfare anyway increases. This is driven by the mitigating impact that consolidation has on the overall search frictions in the market.

3.4.2 Uniqueness of equilibrium

In the previous section we have characterized an equilibrium where the potentially merging firms gain market prominence if they indeed decide to merge. In the equilibrium or Proposition 3.1, consumers found it optimal to first search the products of the merged entity to continue later, if desired, searching the products of the non-merging firms. For this equilibrium to exist, search costs had to be sufficiently



(a) Profits (b) Consumer surplus

Figure 3.2. Pre- and post-merger profits and consumer surplus ($n = 5, k = 2$).

large. In this section we argue that the equilibrium in Proposition 3.1 is unique when search costs are indeed relatively high. We prove this by contradiction.

Suppose that consumers find it optimal to start searching for a satisfactory good among the products of the non-merged firms. If this is so, then the reservation utility at the merged entity, $\overline{\overline{x}} - \hat{p}^*$, must be lower than the reservation utility at a non-merging firm, $\bar{x} - \tilde{p}^*$ (where $\bar{x}$ and $\overline{\overline{x}}$, as before, solve (3.1) and (3.6), respectively). In what follows we derive the payoff functions of merged and non-merged firms under this assumption. We then compute the pair of equilibrium prices and show that, for those prices, when search costs are high, the reservation utility at the merged entity would be above the reservation utility at the non-merged firms, which constitutes a contradiction.

As before, let $\tilde{p}^*$ and $\hat{p}^*$ be the equilibrium prices and assume $\bar{x} - \tilde{p}^* > \overline{\overline{x}} - \hat{p}^*$. Invoking Weitzman (1979) again, given this inequality, consumers should start their search by visiting the non-merged firms. They will visit the merged entity only after having visited all the non-merged shops.

We first derive the payoff of a (deviant) non-merging firm. Consider a non-merging firm, say firm i, that deviates by charging a price $\tilde{p} \neq \tilde{p}^*$. As all non-merging firms are supposed to charge the same price $\tilde{p}^*$, consumers are assumed to visit them randomly. The deviant firm may be visited by the consumer in the first place, the second place, etc. all the way till the $(n-k)$-th place. The probability that the deviating firm is visited in any of these positions is $1/(n-k)$. When the consumer visits the deviant firm in the 1st, 2nd, ..., $(n-k-1)$-th place, the decision whether to continue searching or not takes into account that the next visited shop is also a non-merging store. By contrast, when the deviant firm is the last non-merging store visited by the consumer, i.e. the $(n-k)$-th one, the decision of the consumer is slightly different because the next shop to be visited is a merging store and such a store charges a price different from the price of a non-merging store. Since the consumer stopping rule at any of the first $n-k-1$ non-merging stores is different from the one at the last non-merging store, it is convenient to distinguish among those two cases.

Consider a deviant firm i charging price $\tilde{p}$ and visited in positions $1, 2, ...n-k-1$. This type of firm receives demand from three types of consumers: one, from consumers who visit firm i and terminate their search there; two, from consumers who walk away from all non-merging firms including firm i and return to it after all; and three, from consumers who walk away from all (non-merging and merging) firms in the market and return to it in the end. We now compute the sizes of these groups of consumers. First, a consumer terminates her search immediately after visiting firm i if $\varepsilon_i > \bar{x} - \tilde{p}^* + \tilde{p}$. Therefore, conditional on reaching firm i, the probability that a consumer buys in shop i without searching further equals

$$1 - \bar{x} + \tilde{p}^* - \tilde{p}. \tag{3.16}$$

The consumer may have visited other non-merging firms before arriving at shop i. Since all other firms charge equilibrium prices, if a consumer has walked away from other shops this means that the match values there are lower than $\bar{x}$. Therefore, the demand received by the deviant firm from consumers who visit it in positions 1 to $n-k-1$ and terminate their search right away is

$$\frac{1}{n-k}\sum_{j=1}^{n-k-1}\bar{x}^{j-1}\left(1-\bar{x}+\tilde{p}^*-\tilde{p}\right) = \frac{1}{n-k}\frac{1-\bar{x}^{n-k-1}}{1-\bar{x}}\left(1-\bar{x}+\tilde{p}^*-\tilde{p}\right) \tag{3.17}$$

We now compute the demand received by this type of firm from consumers who walk away from all non-merging firms including firm i and return to it after all. Consider a consumer who is at the last non-merging firm and is therefore contemplating whether to continue searching for a satisfactory good among the products of the merged entity. The consumer will continue searching if the highest match value observed so far is lower than $\bar{\bar{x}} - \hat{p}^* + \tilde{p}^*$. Therefore, the consumer will return to the deviant firm i without checking the products of the merged entity with probability:

$$\Pr\left[\bar{\bar{x}} - \hat{p}^* < \varepsilon_i - \tilde{p} < \bar{x} - \tilde{p}^* \text{ and } \varepsilon_i - \tilde{p} > z_{n-k-1} - \tilde{p}^* \text{ and } z_{n-k-1} \leq \bar{x}\right]$$

$$= \int_{\bar{\bar{x}}-\hat{p}^*+\tilde{p}}^{\bar{x}-\tilde{p}^*+\tilde{p}} (\varepsilon - \tilde{p} + \tilde{p}^*)^{n-k-1} d\varepsilon = \frac{1}{n-k}\left(\bar{x}^{n-k} - \left(\bar{\bar{x}} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\right) \quad (3.18)$$

Finally, we calculate the demand received by firm i from consumers who visit all sellers in the market and return to firm i to buy there. The probability of this event equals

$$\begin{aligned} &\Pr\left[0 < \varepsilon_i - \tilde{p} < \bar{\bar{x}} - \hat{p}^* \text{ and } \varepsilon_i - \tilde{p} > \max\left\{z_{n-k-1} - \tilde{p}^*, z_k - \hat{p}^*\right\}\right. \\ &\left.\text{and } z_{n-k-1} - \tilde{p}^* \leq \bar{\bar{x}} - \hat{p}^*\right] = \int_0^{\bar{\bar{x}}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k d\varepsilon \end{aligned} \quad (3.19)$$

Note that the number of these consumers does not depend on the position in which firm i has been visited.

As a result, we conclude that the total demand of firm i if it is sampled in position 1 to $n-k-1$ equals to the sum of (3.17), (3.18) and (3.19).

We now move to consider the demand of firm i when it is the last non-merging firm visited by the consumer. This type of firm receives demand from two types of consumers: one, consumers who stop searching at firm i right away: and two, consumers who visit all the shops in the market and return to firm i after all. The probability that a consumer terminates her search at firm i without visiting the merged entity equals:

$$\begin{aligned} &\Pr\left[z_{n-k-1} < \bar{x} \text{ and } \ \varepsilon_i - \tilde{p} > \bar{\bar{x}} - \hat{p}^* \text{ and} \varepsilon_i - \tilde{p} > z_{n-k-1} - \tilde{p}^*\right] \\ &= \frac{1}{n-k}\bar{x}^{n-k-1}(1 - \bar{x} + \tilde{p}^* - \tilde{p}) + \frac{1}{n-k}\left(\bar{x}^{n-k} - \left(\bar{\bar{x}} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\right) \end{aligned} \quad (3.20)$$

The returning demand of firm i does not depend on the order of sampling and is identical to (3.19).

Putting the above demands together, the total payoff of a deviating non-merging firm equals:

$$\begin{aligned} \tilde{\pi}(\tilde{p}) = \tilde{p} &\left[\frac{1}{n-k}\frac{1-\bar{x}^{n-k}}{1-\bar{x}}(1 - \bar{x} + \tilde{p}^* - \tilde{p}) + \frac{1}{n-k}\left(\bar{x}^{n-k} - \left(\bar{\bar{x}} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\right)\right. \\ &\left.+ \int_0^{\bar{\bar{x}}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k d\varepsilon\right] \end{aligned}$$

Taking the FOC, imposing the rational expectations requirement that $\tilde{p} = \tilde{p}^*$ and simplifying gives:

$$\frac{1}{n-k}\frac{1-\bar{x}^{n-k}}{1-\bar{x}}\left(1-\bar{x}-\tilde{p}^*\right)+\frac{1}{n-k}\left(\bar{x}^{n-k}-\left(\bar{\bar{x}}-\hat{p}^*+\tilde{p}^*\right)^{n-k}\right)$$
$$+\int_0^{\bar{\bar{x}}-\hat{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k-1}\left(\varepsilon+\hat{p}^*\right)^k d\varepsilon=0 \tag{3.21}$$

We now derive the payoff of the merged entity when it deviates to $\hat{p} \neq \hat{p}^*$. As discussed above, consumers will walk away from all non-merging stores and arrive at the merged entity if the best of the non-merging firms' deals is lower than $\bar{\bar{x}}-\hat{p}^*$. Therefore, the merged entity will only receive demand when its offer is the best of all the offers in the market. The probability of this event equals

$$\Pr\left[z_{n-k}-\tilde{p}^*<\bar{\bar{x}}-\hat{p}^* \text{ and } z_k-\hat{p}>\max\left\{0;\, z_{n-k}-\tilde{p}^*\right\}\right]$$
$$=\left(\bar{\bar{x}}-\hat{p}^*+\tilde{p}^*\right)^{n-k}\left(1-\left(\bar{\bar{x}}-\hat{p}^*+\hat{p}\right)^k\right)+k\int_{\hat{p}}^{\bar{\bar{x}}-\hat{p}^*-\hat{p}}\left(\varepsilon-\hat{p}+\tilde{p}^*\right)^{n-k}\varepsilon^{k-1}d\varepsilon$$

Its payoff then equals:

$$\hat{\pi}(\hat{p})=\hat{p}\left[\left(\bar{\bar{x}}-\hat{p}^*+\tilde{p}^*\right)^{n-k}\left(1-\left(\bar{\bar{x}}-\hat{p}^*+\hat{p}\right)^k\right)\right.$$
$$\left.+\,k\int_0^{\bar{\bar{x}}-\hat{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k}\left(\varepsilon+\hat{p}\right)^{k-1}d\varepsilon\right]$$

Taking the FOC, imposing the rational expectations condition that $\hat{p}=\hat{p}^*$ and simplifying gives:

$$\left(\bar{\bar{x}}-\hat{p}^*+\tilde{p}^*\right)^{n-k}\left(1-\bar{\bar{x}}^k-k\bar{\bar{x}}^{k-1}\hat{p}^*\right)$$
$$=k\int_0^{\bar{\bar{x}}-\hat{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+k\hat{p}^*\right)d\varepsilon=0 \tag{3.22}$$

Proposition 3.4. *Assume that a k-firm merger takes place in the market. Assume also that the search cost is sufficiently high. Then an equilibrium where consumers prefer to search first the products of the non-merging firms and then, if they wish so, the products of the merged entity does not exist. As a result, the equilibrium in Proposition 3.1 is unique.*

Proof. Assume that there is a pair of non-negative prices $\hat{p}^*$ and $\tilde{p}^*$ that satisfy the system of equations (3.21) and (3.22). For these prices to be consistent with equilibrium, first, they must be lower than or equal to the monopoly prices p_k^m and p^m, respectively; moreover, the reservation utility at the merged entity must be lower than the reservation utility at a non-merging firm.

Take the LHS of the FOC of a non-merging firm, equation (3.21). Note that the integral in this equation is positive. Observe now that the second summand, $\frac{1}{n-k}\left(\bar{x}^{n-k} - \left(\bar{\bar{x}} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\right)$, is also positive because the assumption $\bar{x} - \tilde{p}^* > \bar{\bar{x}} - \hat{p}^*$ implies that $\bar{x} > \bar{\bar{x}} - \hat{p}^* + \tilde{p}^*$. As a consequence, if an equilibrium exists, the first term of the FOC (3.21) must be negative. This implies that in equilibrium, it must be the case that $\tilde{p}^* > 1 - \bar{x}$.

Take now the limiting case where the search cost is high so that $\bar{x} \to 1/2$. If this is so, for an equilibrium to exist, it must be the case that $\tilde{p}^* > 1/2$. But this is a contradiction because $\tilde{p}^* \leq p^m = 1/2$. ■

3.5 Conclusions

In the long-run firms that merge end up shutting down shops and clustering their products together. In this paper we have argued that when search costs are significant, this process generates substantial demand-side economies. What happens is that in the post-merger market, everything else equal, consumers do not need to search as intensively as in the pre-merger situation to find satisfactory products. This paper has emphasized the importance of these demand-side economies. We have shown that firms that merge may gain a prominent position in the market, even if they increase their prices more than what the non-merging firms do. In that case, consumers prefer to start searching for satisfactory products at the merged entity. In equilibrium, insider firms gain customers and increase their profits, while outsider firms lose out because they are pushed all the way back in the optimal search order that consumers follow when they search for products. We have shown that consolidation may create sufficiently large search economies so as to generate rents for consumers too.

The difference between the short- and the long-run effects of mergers is important. In a previous chapter, we have studied the short-run implications of mergers. We have shown that firms that merge raise their prices more than what the non-merging firms do, so absent any other offsetting effect, merging firms are pushed all the way back in the optimal search order that consumers follow when they search for satisfactory products. Thus, merging gives the outsiders a free ride that is freer the greater the search costs. For sufficiently large search costs, it turns out that merging in a product differentiation environment lowers the short-run profits of the merging firms. To counter this effect, firms may cluster their products together.

In this chapter we have shown that by doing so search economies unfold, which makes mergers profitable for the merging firms and, sometimes, for the consumers too.

In a recent paper on mergers in the Italian banking sector, Focarelli and Panetta (2003) forcefully make the point that mergers may generate efficiency gains (headquarter consolidation, shutting down of branches, etc.) that take a relatively long time to materialize. In fact, they show that consolidation leads to adverse price changes only temporarily, and that when sufficient time elapses efficiency gains kick-in and prices decrease. Seen against earlier empirical results by Kim and Singal (1993) and Prager and Hannan (1998), this is an interesting insight. Our theory points out that after-merger downsizing may lead to important search economies that in the long-run may result in price decreases. In this sense it yields implications consistent with Focarelli and Panetta (2003)'s empirical findings. Whether supply- or demand-side economies cause the results remains an empirical question. Developing methods to separate the relative importance of demand- versus cost-economies seems a fascinating area of research.

3.A Appendix

Proof of Proposition 3.1. The proof of the proposition is organized in three claims. The first claim shows that there is a pair of prices $\hat{p}^*$ and $\tilde{p}^*$ that satisfy the first order conditions (3.11) and (3.12). The second claim shows that this pair is unique, and the last claim shows that it is optimal for consumers to start searching from a merger if the search costs are sufficiently high.

Let $G(\hat{p}^*, \tilde{p}^*)$ and $H(\hat{p}^*, \tilde{p}^*)$ denote the LHS of the FOCs (3.11) and (3.12), respectively. In what follows, we drop the "$*$" super-indexes to shorten the expressions.

Claim 3.A.1. *There is a pair of prices $\hat{p}$ and $\tilde{p}$ that satisfy the first-order conditions $G(\hat{p}, \tilde{p}) = 0$ and $H(\hat{p}, \tilde{p}) = 0$.*

Proof. The function G is differentiable and takes on real values for all $(\hat{p}, \tilde{p}) \in \left[0, p_k^m\right] \times [0, p^m]$. Therefore, the FOC $G(\tilde{p}, \hat{p}) = 0$ defines an implicit relation between $\hat{p}$ and $\tilde{p}$. Let us denote such relationship by $\tilde{p} = v_1(\hat{p})$. We now argue that v_1 is increasing. By the implicit function theorem

$$\frac{\partial v_1}{\partial \hat{p}} = \frac{-\partial G/\partial \hat{p}}{\partial G/\partial \tilde{p}} \tag{3.A.1}$$

We next note that G is decreasing in $\hat{p}$ and increasing in $\tilde{p}$. To see this, compute first

$$\begin{aligned}
\frac{\partial G}{\partial \hat{p}} &= \Big[-(k-1)\left(\overline{x} - \tilde{p} + \hat{p}\right)^{k-2}\left(\overline{x} - \tilde{p} + (k+1)\,\hat{p}\right) - (k+1)\left(\overline{x} - \tilde{p} + \hat{p}\right)^{k-1} \\
&\quad + k\,(k-1)\int_0^{\overline{x}-\tilde{p}} \left(\varepsilon + \tilde{p}\right)^{n-k}\left(\varepsilon + \hat{p}\right)^{k-3}\left(2\varepsilon + k\hat{p}\right) d\varepsilon\Big] \\
&= -k\left(\overline{x} - \tilde{p} + \hat{p}\right)^{k-2}\left(2\overline{x} - 2\tilde{p} + (k+1)\,\hat{p}\right) \\
&\quad + k\,(k-1)\int_0^{\overline{x}-\tilde{p}} \left(\varepsilon + \tilde{p}\right)^{n-k}\left(\varepsilon + \hat{p}\right)^{k-3}\left(2\varepsilon + k\hat{p}\right) d\varepsilon
\end{aligned}$$

Note next that $\partial G/\partial \hat{p}$ decreases in $\bar{x}$. This is because

$$\begin{aligned}
\frac{1}{k}\frac{\partial^2 G}{\partial \overline{x}\partial \hat{p}} &= -(k-2)\left(\overline{x} - \tilde{p} + \hat{p}\right)^{k-3}\left(2\overline{x} - 2\tilde{p} + (k+1)\,\hat{p}\right) - 2\left(\overline{x} - \tilde{p} + \hat{p}\right)^{k-2} \\
&\quad + (k-1)\,\overline{x}^{n-k}\left(\overline{x} - \tilde{p} + \hat{p}\right)^{k-3}\left(2\overline{x} - 2\tilde{p} + k\hat{p}\right) \\
&= -(k-1)(\overline{x} - \tilde{p} + \hat{p})^{k-3}(2\overline{x} - 2\tilde{p} + k\hat{p})\left(1 - \overline{x}^{n-k}\right) < 0
\end{aligned}$$

We know that $\overline{x} \geq \tilde{p}$. If we evaluate $\partial G/\partial \hat{p}$ at $\overline{x} = \tilde{p}$ we obtain $\partial G/\partial \hat{p} = -k\,(k+1)\,\hat{p}^{k-1} <$

0. Since $\partial G/\partial \hat{p}$ decreases in $\bar{x}$, then we conclude $\partial G/\partial \hat{p}$ is negative for all $\overline{x}$.

Compute now

$$\begin{aligned}
\frac{\partial G}{\partial \tilde{p}} &= (k-1)\left(\overline{x}-\tilde{p}+\hat{p}\right)^{k-2}\left(\overline{x}-\tilde{p}+(k+1)\,\hat{p}\right)+\left(\overline{x}-\tilde{p}+\hat{p}\right)^{k-1} \\
&+k\,(n-k)\int_0^{\overline{x}-\tilde{p}}\left(\varepsilon+\tilde{p}\right)^{n-k-1}\left(\varepsilon+\hat{p}\right)^{k-2}\left(\varepsilon+k\hat{p}\right)d\varepsilon \\
&-k\overline{x}^{n-k}\left(\overline{x}-\tilde{p}+\hat{p}\right)^{k-2}\left(\overline{x}-\tilde{p}+k\hat{p}\right) \\
&=k\left(\overline{x}-\tilde{p}+\hat{p}\right)^{k-2}\left(\overline{x}-\tilde{p}+k\hat{p}\right)\left(1-\overline{x}^{n-k}\right) \\
&+k\,(n-k)\int_0^{\overline{x}-\tilde{p}}\left(\varepsilon+\tilde{p}\right)^{n-k-1}\left(\varepsilon+\hat{p}\right)^{k-2}\left(\varepsilon+k\hat{p}\right)d\varepsilon>0
\end{aligned}$$

As a consequence, v_1 is increasing in $\hat{p}$.

We now observe that the solution of the equation $G(\hat{p},\tilde{p})=0$ when $\hat{p}=0$ is negative. We establish this by contradiction. Suppose that the solution to $G(0,\tilde{p})=0$ is some non-negative number. If this is so, since we know G increases in $\tilde{p}$, it should be the case that $G(0,0)<0$. However,

$$G(0,0)=1-\overline{x}^k+k\int_0^{\overline{x}}\varepsilon^{n-1}>0,$$

which leads to a contradiction. Summarizing, we have shown that the implicit function v_1, defined on $[0,p_k^m]$, starts taking negative values and is increasing.

Consider now the second FOC $H(\hat{p},\tilde{p})=0$ and rewrite it as

$$\frac{1-\overline{x}^{n-k}}{(n-k)\,(1-\overline{x})}\left(1-\overline{x}-\tilde{p}\right)+\frac{1}{\left(\overline{x}-\tilde{p}+\hat{p}\right)^k}\int_0^{\overline{x}-\tilde{p}}\left(\varepsilon+\hat{p}\right)^k\left(\varepsilon+\tilde{p}\right)^{n-k-1}d\varepsilon=0$$

Let us denote the LHS of this expression by $L(\hat{p},\tilde{p})$. The equation $L(\hat{p},\tilde{p})=0$ defines an implicit relationship between $\hat{p}$ and $\tilde{p}$, which we denote $\tilde{p}=v_2(\hat{p})$. We show next v_2 is also increasing. By the implicit function theorem we have

$$\frac{\partial v_2}{\partial \hat{p}}=\frac{-\partial L/\partial \hat{p}}{\partial L/\partial \tilde{p}},$$

We note that L is increasing in $\hat{p}$ and decreasing in $\tilde{p}$. The first observation comes from

$$\frac{\partial L}{\partial \hat{p}}=\frac{k}{\left(\overline{x}-\tilde{p}+\hat{p}\right)^{k+1}}\int_0^{\overline{x}-\tilde{p}}\left(\varepsilon+\hat{p}\right)^{k-1}\left(\varepsilon+\tilde{p}\right)^{n-k-1}\left(\overline{x}-\tilde{p}-\varepsilon\right)d\varepsilon>0.$$

For the second, we compute

$$\frac{\partial L}{\partial \tilde{p}} = -\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})} + \frac{k}{(\overline{x}-\tilde{p}+\hat{p})^{k+1}} \int_0^{\overline{x}-\tilde{p}} (\varepsilon+\hat{p})^k (\varepsilon+\tilde{p})^{n-k-1} d\varepsilon$$
$$+ \frac{n-k-1}{(\overline{x}-\tilde{p}+\hat{p})^k} \int_0^{\overline{x}-\tilde{p}} (\varepsilon+\hat{p})^k (\varepsilon+\tilde{p})^{n-k-2} d\varepsilon - \overline{x}^{n-k-1} \quad (3.A.2)$$

It is difficult to evaluate the sign of this derivative on inspection. To ease the evaluation, consider first the term in the second line of this derivative. We note that

$$\frac{n-k-1}{(\overline{x}-\tilde{p}+\hat{p})^k} \int_0^{\overline{x}-\tilde{p}} (\varepsilon+\hat{p})^k (\varepsilon+\tilde{p})^{n-k-2} d\varepsilon - \overline{x}^{n-k-1}$$
$$< (n-k-1) \int_0^{\overline{x}-\tilde{p}} (\varepsilon+\tilde{p})^{n-k-2} d\varepsilon - \overline{x}^{n-k-1} = -\tilde{p}^{n-k-1} < 0 \quad (3.A.3)$$

Consider next the first term of (3.A.2) and note that

$$\frac{k}{(\overline{x}-\tilde{p}+\hat{p})^{k+1}} \int_0^{\overline{x}-\tilde{p}} (\varepsilon+\hat{p})^k (\varepsilon+\tilde{p})^{n-k-1} d\varepsilon - \frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}$$
$$< \frac{k\overline{x}^{n-k-1}}{(\overline{x}-\tilde{p}+\hat{p})^{k+1}} \int_0^{\overline{x}-\tilde{p}} (\varepsilon+\hat{p})^k d\varepsilon - \frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}$$
$$= \frac{k\bar{x}^{n-k-1}}{k+1} - \frac{k\bar{x}^{n-k-1}\hat{p}^{k+1}}{(k+1)(\overline{x}-\tilde{p}+\hat{p})^{k+1}} - \frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}$$
$$< \frac{k\bar{x}^{n-k-1}}{k+1} - \frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})} = \frac{1}{1-\bar{x}} \left[\frac{k\bar{x}^{n-k-1}(1-\bar{x})}{k+1} - \frac{1-\bar{x}^{n-k}}{n-k} \right] \quad (3.A.4)$$

We now argue that the term in square brackets in the last line of (3.A.4) is negative for all $\bar{x}$. To see this, we first observe that it increases in $\bar{x}$. In fact, taking the derivative w.r.t. $\bar{x}$ we get

$$\frac{k+1}{\bar{x}^{n-k-2}} \frac{\partial}{\partial \bar{x}} \left[\frac{k\bar{x}^{n-k-1}(1-\bar{x})}{k+1} - \frac{1-\bar{x}^{n-k}}{n-k} \right] = k(n-k-1) - k(n-k)\bar{x}$$
$$+ (k+1)\bar{x} = -k(k+1) + (k+1)\bar{x} + k^2\bar{x} + nk(1-\bar{x})$$
$$\geq -k(k+1) + (k+1)\bar{x} + k^2\bar{x} + (k+1)k(1-\bar{x}) = \bar{x} > 0$$

Since for the highest possible $\bar{x}$ we have

$$\lim_{\bar{x}\to 1} \frac{k\bar{x}^{n-k-1}(1-\bar{x})}{k+1} - \frac{1-\bar{x}^{n-k}}{n-k} = 0$$

we conclude that (3.A.4) is negative. This in turn implies that L decreases in $\tilde{p}$. Since L is increasing in $\hat{p}$ and decreasing in $\tilde{p}$, the function v_2, defined implicitly by the first order condition $H(\hat{p}, \tilde{p}) = 0$, is also increasing in $\hat{p}$.

We finally observe that the solution to $L(\hat{p}, \tilde{p}) = 0$ when $\hat{p} = 0$ must be a positive number. By contradiction, suppose that the solution to $L(0, \tilde{p}) = 0$ is some negative number. If this is so, since we know L decreases in $\tilde{p}$, it should be the case that $L(0,0) < 0$. However,

$$L(0,0) = \frac{1-\overline{x}^{n-k}}{n-k} + \frac{1}{\overline{x}^k}\int_0^{\overline{x}-\tilde{p}} \varepsilon^{n-1} d\varepsilon > 0,$$

which constitutes a contradiction. Summarizing, we have now shown that the implicit function v_2 defined on $[0, p_k^m]$ starts taking positive values and is increasing.

To show that v_1 and v_2 cross at least once, we now prove that $v_1\left(p_k^m\right) = \overline{x} > v_2\left(p_k^m\right)$ (since both are increasing in $\tilde{p}$ and we know that $v_1(0) < 0 < v_2(0)$). Setting $\hat{p} = p_k^m$ in the FOC for the merged entity gives

$$\begin{aligned} G\left(p_k^m, \tilde{p}\right) = {} & 1 - \left(\overline{x} - \tilde{p} + p_k^m\right)^{k-1}\left(\overline{x} - \tilde{p} + (k+1)\, p_k^m\right) \\ & + k \int_0^{\overline{x}-\tilde{p}} (\varepsilon + \tilde{p})^{n-k} \left(\varepsilon + p_k^m\right)^{k-2} \left(\varepsilon + k p_k^m\right) d\varepsilon = 0 \end{aligned}$$

which solution is $\tilde{p} = \overline{x}$ since $G\left(p_k^m, \overline{x}\right) = 1 - (k+1)\left(p_k^m\right)^k = 0$ by definition of p_k^m.

Likewise setting $\hat{p} = p_k^m$ in the FOC for the non-merging firm gives

$$\begin{aligned} L\left(p_k^m, \tilde{p}\right) = {} & \frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}\left(1 - \overline{x} - \tilde{p}\right) \\ & + \frac{1}{\left(\overline{x} - \tilde{p} + p_k^m\right)^k} \int_0^{\overline{x}-\tilde{p}} \left(\varepsilon + p_k^m\right)^k (\varepsilon + \tilde{p})^{n-k-1} d\varepsilon = 0 \end{aligned}$$

Since $L\left(p_k^m, \overline{x}\right) = \left(1 - \overline{x}^{n-k}\right)\left(1 - 2\overline{x}\right) / (n-k)(1-\overline{x}) \leq 0$ and we know that L decreases in $\tilde{p}$, it is clear that the solution to $L\left(p_k^m, \tilde{p}\right) = 0$ must be some $\tilde{p} < \overline{x}$.

To complete the proof of existence, it remains to be shown that at the point(s) at which v_1 and v_2 cross we have $\tilde{p} \leq p^m = 1/2$. For this, it suffices to show that $L\left(p_k^m, 1/2\right) < 0$ because since L decreases in $\tilde{p}$, this means that the solution to $L\left(p_k^m, \tilde{p}\right) = 0$ must be some $\tilde{p} < 1/2$. In fact, setting $\tilde{p} = 1/2$, we get

$$L\left(p_k^m, 1/2\right) = \frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}\left(\frac{1}{2} - \overline{x}\right)$$

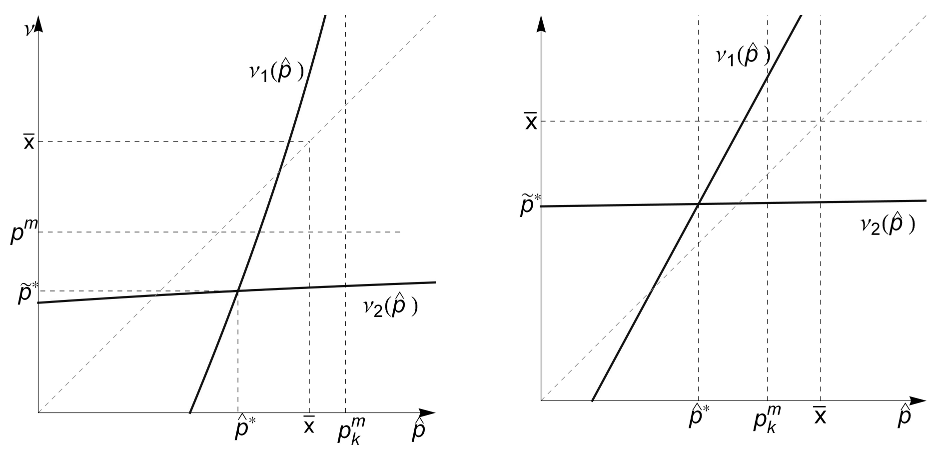
$$+\frac{1}{\left(\overline{x}-\frac{1}{2}+p_k^m\right)^k}\int_0^{\overline{x}-\frac{1}{2}}\left(\varepsilon+p_k^m\right)^k\left(\varepsilon+\frac{1}{2}\right)^{n-k-1}d\varepsilon. \tag{3.A.5}$$

We now note that $L\left(p_k^m, 1/2\right)$ decreases in $\overline{x}$. To see this, compute

$$\begin{aligned}\frac{\partial L\left(p_k^m,\frac{1}{2}\right)}{\partial\overline{x}} &= -\frac{1+(n-k-1)\overline{x}^{n-k}-(n-k)\overline{x}^{n-k-1}}{2(n-k)(1-\overline{x})^2}\\ &\quad-\frac{k}{\left(\overline{x}-\frac{1}{2}+p_k^m\right)^{k+1}}\int_0^{\overline{x}-1/2}\left(\varepsilon+p_k^m\right)^k\left(\varepsilon+\frac{1}{2}\right)^{n-k-1}d\varepsilon \end{aligned}\tag{3.A.6}$$

and notice that $1+(n-k-1)\overline{x}^{n-k}-(n-k)\overline{x}^{n-k-1}>0$ for all $\overline{x}$ (since it decreases in $\overline{x}$ and equals zero when $\overline{x}=1$). Therefore, if $L\left(p_k^m, 1/2\right)\le 0$ for the lowest value of $\overline{x}$, then it is negative everywhere. In fact, setting $\overline{x}=1/2$ in (3.A.5) yields $L\left(p_k^m, 1/2\right)=0$. To summarize, we have now shown that v_1 and v_2 cross at least once on $[0, p_k^m]\times[0, p^m]$ so a candidate equilibrium exists.

The arguments in the proof can be seen in Figure 3.A.1. As explained in the main text, the equilibrium candidate may entail prices for the non-merging firms above or below the price of the merged entity. In Figure 3.A.1a, we set $n=3$, $k=2$ and $\overline{x}=0.54$ so that the internalization-of-pricing-externalities effect dominates the search-order effect. In Figure 3.A.1b we depict a case with many firms ($n=10$, $k=2$ and $\overline{x}=0.9$) where the search-order effect has a dominating influence over the internalization-of-pricing-externalities effect. *QED*



(a) Case $\hat{p}^*>\tilde{p}^*$ (b) Case $\hat{p}^*<\tilde{p}^*$

Figure 3.A.1. Existence of equilibrium

Claim 3.A.2. *The pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies (3.11) and (3.12) is unique.*

Proof. We start by noting that v_1 is increasing in $\hat{p}$ at a rate greater than 1. Using the derivations above, this follows from the following remarks. First, note that

$$\begin{aligned}\frac{1}{k}\left(-\frac{\partial G}{\partial \hat{p}}-\frac{\partial G}{\partial \tilde{p}}\right) &= (\overline{x}-\tilde{p}+\hat{p})^{k-2}\left((\overline{x}-\tilde{p}+\hat{p})+\overline{x}^{n-k}(\overline{x}-\tilde{p}+k\hat{p})\right)\\ &\quad-(k-1)\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\tilde{p})^{n-k}(\varepsilon+\hat{p})^{k-3}(2\varepsilon+k\hat{p})d\varepsilon\\ &\quad-(n-k)\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\tilde{p})^{n-k-1}(\varepsilon+\hat{p})^{k-2}(\varepsilon+k\hat{p})d\varepsilon. \qquad (3.A.7)\end{aligned}$$

Observe now that this expression is increasing in $\overline{x}$, as its derivative with respect to $\overline{x}$ equals $(k-1)(\overline{x}-\tilde{p}+\hat{p})^{k-2}\left(1-\overline{x}^{n-1}\right)\geq 0$. Therefore, if (3.A.7) is positive when $\overline{x}$ takes on its lowest value, then it is positive everywhere. Setting $\overline{x}=\tilde{p}$ in the RHS of (3.A.7) gives $\hat{p}^{k-2}\left(\hat{p}+k\tilde{p}^{n-k}\hat{p}\right)>0$, which proves that v_1 increases with slope greater than 1.

We continue by noting that the rate at which v_2 increases is lower than 1. Using the derivations above, since $\partial L/\partial\tilde{p}<0$, we need to show that

$$\begin{aligned}\frac{\partial L}{\partial \hat{p}}+\frac{\partial L}{\partial \tilde{p}} &= -\frac{1-\overline{x}^{n-k}}{(n-k)\,(1-\overline{x})}+\frac{k}{(\overline{x}-\tilde{p}+\hat{p})^k}\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k-1}(\varepsilon+\tilde{p})^{n-k-1}\,d\varepsilon\\ &\quad+\frac{n-k-1}{(\overline{x}-\tilde{p}+\hat{p})^k}\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k}(\varepsilon+\tilde{p})^{n-k-2}\,d\varepsilon-\overline{x}^{n-k-1} \qquad (3.A.8)\end{aligned}$$

is negative. Now notice that the last line of this expression is negative (from (3.A.3)). Moreover, regarding the first line of (3.A.8) we have

$$\begin{aligned}&-\frac{1-\overline{x}^{n-k}}{(n-k)\,(1-\overline{x})}+\frac{k}{(\overline{x}-\tilde{p}+\hat{p})^k}\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k-1}(\varepsilon+\tilde{p})^{n-k-1}\,d\varepsilon\\ &<-\frac{1-\overline{x}^{n-k}}{(n-k)\,(1-\overline{x})}+\frac{k\overline{x}^{n-k-1}}{(\overline{x}-\tilde{p}+\hat{p})^k}\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k-1}\,d\varepsilon\\ &=-\frac{1-\overline{x}^{n-k}}{(n-k)\,(1-\overline{x})}+\frac{\overline{x}^{n-k-1}}{(\overline{x}-\tilde{p}+\hat{p})^k}\left[(\overline{x}-\tilde{p}+\hat{p})^k-\hat{p}^k\right]\\ &<-\frac{1-\overline{x}^{n-k}}{(n-k)\,(1-\overline{x})}+\overline{x}^{n-k-1}=-\frac{1+(n-k-1)\,\bar{x}^{n-k}-(n-k)\,\bar{x}^{n-k-1}}{(n-k)\,(1-\bar{x})}<0,\end{aligned}$$

where the last inequality follows from the remarks after equation (3.A.6). This implies that v_2 increases at a rate less than 1. This, together with the arguments before shows that there exists a unique candidate equilibrium. *QED*

Claim 3.A.3. *The candidate equilibrium where consumers start searching by the merged entity, which charges a price $\hat{p}^*$, and then continue by the non-merging stores, which charge a price $\widetilde{p}^*$ exists if the search cost s is sufficiently large, in which case $\hat{p}^* \geq \widetilde{p}^*$.*

Proof. Basically, we need to prove that the putative search order which prescribes consumers to go first to the merged entity and then continue searching the non-merging stores is indeed optimal. For this, as described in the main text of the paper, we need to show that $\overline{\overline{x}} - \widehat{p}^* > \overline{x} - \tilde{p}^*$.

(i) Consider the case in which s is sufficiently large, so $s \to 1/8$ and $\bar{x} \to 1/2$. It takes a few derivations to check that the solution to the first order conditions

$$
\begin{aligned}
&1 - \left(\frac{1}{2} - \tilde{p}^* + \hat{p}^*\right)^{k-1} \left(\frac{1}{2} - \tilde{p}^* + (k+1)\,\hat{p}^*\right) \\
&+ k \int_0^{\frac{1}{2}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-2} (\varepsilon + k\hat{p}^*)\, d\varepsilon = 0 \\
&\frac{1}{n-k} \left(\frac{1}{2} - \tilde{p}^* + \hat{p}^*\right)^{k} 2 \left(1 - \frac{1}{2^{n-k}}\right) \left(\frac{1}{2} - \tilde{p}^*\right) \\
&+ \int_0^{\frac{1}{2}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^{k}\, d\varepsilon = 0
\end{aligned}
$$

is

$$
\begin{aligned}
\tilde{p}^* &= p^m = 1/2 = \bar{x} \\
\hat{p}^* &= p_m^k = 1/(1+k)^{\frac{1}{k}}
\end{aligned} \tag{3.A.9}
$$

Inspection of (3.A.9) reveals that when $s \to 1/8$, $\hat{p}^* > \tilde{p}^*$. Moreover, to prove the claim it suffices to show that when $s \to 1/8$

$$\overline{\overline{x}} - \hat{p}^* > 0.$$

From equation (3.6) we know $\overline{\overline{x}}$ satisfies $\int_x^1 (\varepsilon - x) d\varepsilon^k - s = 0$, or

$$\frac{k(1-\overline{\overline{x}}) - \overline{\overline{x}}(1-\overline{\overline{x}}^k)}{k+1} - s = 0. \tag{3.A.10}$$

We note now $\overline{\overline{x}}$ increases in k. This is because the derivative of the LHS of (3.A.10) is

$$\frac{1 + \overline{\overline{x}}^{k+1}((k+1)\ln[\overline{\overline{x}}] - 1)}{(k+1)^2} > 0$$

(the positive sign follows from noting that this derivative decreases in $\overline{\overline{x}}$ and that at $\overline{\overline{x}} = 1$ it takes value zero). Equation (3.A.10) can be rewritten as

$$\overline{\overline{x}} = \frac{k + \overline{\overline{x}}^{k+1}}{k+1} - s$$

Deducting $\hat{p}^*$ on both sides of this equality gives

$$\overline{\overline{x}} - \hat{p}^* = \frac{k + \overline{\overline{x}}^{k+1}}{k+1} - \hat{p}^* - s. \tag{3.A.11}$$

From the solution in (3.A.9) we know that when $s \to 1/8$, then $\hat{p}^* \to 1/(1+k)^{\frac{1}{k}}$. As a result, when $s \to 1/8$, equation (3.A.11) gives

$$\overline{\overline{x}} - p_m^k = \frac{k + \overline{\overline{x}}^{k+1}}{k+1} - \frac{1}{(1+k)^{\frac{1}{k}}} - \frac{1}{8} \tag{3.A.12}$$

Note now that the RHS of (3.A.12) increases in $\overline{\overline{x}}$. Setting the lowest admissible value for $\overline{\overline{x}}$, we have

$$\overline{\overline{x}} - p_m^k = \frac{k + (\frac{1}{2})^{k+1}}{k+1} - \frac{1}{(1+k)^{\frac{1}{k}}} - \frac{1}{8} > 0$$

for all $k \leq n-1$. *QED*

The proof of the Proposition is now complete. ■

Proof of Proposition 3.2. (i) We first show that the merging stores increase their profits after the merger. The difference between the profit per product of the merged entity, $\hat{\pi}_i/k$, and the typical pre-merger profit of a firm, π^*, equals:

$$\frac{\hat{\pi}_i}{k} - \pi^* = \frac{\hat{p}^*}{k}\left(1 - (\overline{x} - \tilde{p}^* + \hat{p}^*)^k + k\int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \hat{p}^*)^{k-1}(\varepsilon + \tilde{p}^*)^{n-k} d\varepsilon\right)$$
$$- \frac{p^*}{n}(1 - p^{*n})$$

Since $\hat{p}^*$ is an equilibrium price, then, given the non-merging firm's price, $\hat{\pi}_i(\hat{p}^*)$ is greater than $\hat{\pi}_i(\hat{p})$ for any $\hat{p} \neq \hat{p}^*$. Therefore, replacing $\hat{p}^*$ by $\tilde{p}^*$ gives

$$\frac{\hat{\pi}_i}{k} - \pi^* > \tilde{p}^*\left(\frac{1 - \overline{x}^k}{k} + \frac{1}{n}(\overline{x}^n - \tilde{p}^{*n})\right) - \frac{p^*}{n}(1 - p^{*n})$$

We now note that $\frac{1-\overline{x}^k}{k} = \int_{\overline{x}}^{1} \varepsilon^{k-1} d\varepsilon$ and is decreasing in k. Therefore,

$$\begin{aligned}
&\tilde{p}^* \left(\frac{1-\overline{x}^k}{k} + \frac{1}{n} \left(\overline{x}^n - \tilde{p}^{*n} \right) \right) - \frac{p^*}{n} \left(1 - p^{*n} \right) \\
>&\tilde{p}^* \left(\frac{1-\overline{x}^{n-1}}{n-1} + \frac{1}{n} \left(\overline{x}^n - \tilde{p}^{*n} \right) \right) - \frac{p^*}{n} \left(1 - p^{*n} \right) \\
=&\frac{\tilde{p}^*}{n\left(n-1\right)} \left(n - n\overline{x}^{n-1} + \left(n-1\right) \overline{x}^n - \left(n-1\right) \tilde{p}^{*n} \right) - \frac{p^*}{n} \left(1 - p^{*n} \right)
\end{aligned}$$

where the inequality follows from setting $k = n-1$. Observe next that this expression is decreasing in $\overline{x}$ since its derivative with respect to $\overline{x}$ is equal to $-\overline{x}^{n-2} \left(n(n-1)(1-\overline{x}) \right) < 0$. Therefore we have

$$\begin{aligned}
&\frac{\tilde{p}^*}{n\left(n-1\right)} \left(n - n\overline{x}^{n-1} + \left(n-1\right) \overline{x}^n - \left(n-1\right) \tilde{p}^{*n} \right) - \frac{p^*}{n} \left(1 - p^{*n} \right) \\
>&\frac{\tilde{p}^*}{n\left(n-1\right)} \left(n - 1 - \left(n-1\right) \tilde{p}^{*n} \right) - \frac{p^*}{n} \left(1 - p^{*n} \right) \\
=&\frac{\tilde{p}^*}{n} \left(1 - \tilde{p}^{*n} \right) - \frac{p^*}{n} \left(1 - p^{*n} \right)
\end{aligned}$$

where the inequality follows from setting $\overline{x} = 1$.

Finally, note that the expression $\frac{\tilde{p}^*}{n} \left(1 - \tilde{p}^{*n} \right)$ is increasing in $\tilde{p}^*$ because its derivative with respect to $\tilde{p}^*$ is equal to $\frac{1}{n} \left(1 - \left(n+1\right) \tilde{p}^{*n} \right) > 0$. Therefore, if it is the case that $\tilde{p}^* > p^*$, we can write

$$\frac{\tilde{p}^*}{n} \left(1 - \tilde{p}^{*n} \right) - \frac{p^*}{n} \left(1 - p^{*n} \right) > \frac{p^*}{n} \left(1 - p^{*n} \right) - \frac{p^*}{n} \left(1 - p^{*n} \right) = 0$$

where the inequality follows from replacing $\tilde{p}^*$ with p^*. We then conclude that $\frac{\hat{\pi}_i}{k} - \pi^* > 0$ if $\tilde{p}^* > p^*$.

To complete the argument, we now show $\tilde{p}^* > p^*$ is indeed true. For this, we build on a result of Zhou (2009). Consider the following modification of our model. Suppose that (i) the merging stores did not internalize the price-effects they impose on one another (ii) the merged entity continued to keep all its stores and (iii) consumers visited the merging stores first. If this were so, this modified model would be exactly identical to the market situation studied in Zhou (2009) article on the effects of market prominence. "Prominent" firms are searched first by the consumers, who, in case they do not find a satisfactory product, continue searching among the non-prominent firms.

In such a modified model, the payoff of a deviant (potentially) merging firm (or prominent firm in Zhou's terminology) would then be given by

$$\hat{\pi}_j = \hat{p}_j \left[\frac{1}{k} \frac{1-\bar{x}^k}{1-\bar{x}} \left(1 - \bar{x} + \hat{p}^* - \hat{p}_j\right) + \frac{1}{k} \left(\bar{x}^k - \left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^k \right) \right.$$
$$\left. + \int_0^{\bar{x}-\tilde{p}^*} \left(\varepsilon + \tilde{p}^*\right)^{n-k} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon \right] \tag{3.A.13}$$

where $\hat{p}_j$ is indexed by j to indicate that it is the deviation price of a single potentially merging firm j. In the modified model, however, the payoff to a non-merging firm (or non-prominent firm) is exactly the same as the payoff in our model given in (3.10). This signifies that the reaction function of a non-merging firm is identical to $\nu_2(\hat{p})$, which was derived in the proof of claim 3.A.1.

Taking the FOC in (3.A.13) gives:

$$\frac{1}{k} \frac{1-\bar{x}^k}{1-\bar{x}} \left(1 - \bar{x} - \hat{p}^*\right) + \frac{1}{k} \left(\bar{x}^k - \left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^k \right)$$
$$+ \int_0^{\bar{x}-\tilde{p}^*} \left(\varepsilon + \tilde{p}^*\right)^{n-k} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon = 0 \tag{3.A.14}$$

This equation defines implicitly a relation $\tilde{p} = \nu_0(\hat{p})$. The crossing point between $\nu_0(\hat{p})$ and $\nu_2(\hat{p})$ gives the equilibrium prices $\hat{p}^*$ and $\tilde{p}^*$ in the modified model. As in our original model, note that the functions $\nu_0(\hat{p})$ and $\nu_2(\hat{p})$ can be interpreted as the joint reaction functions of the potentially merging and non-merging firms, respectively.

Zhou (2009) Proposition 3 shows that non-prominent firms charge a price above the price the firms would charge if no firm were prominent. In the jargon of this chapter, this implies that the non-merging firms in this modified model will charge a price above the pre-merger price, which is the result we want to prove.

To complete the proof, we now argue that moving from the modified model to the original model can only enhance the difference between the price of a non-merging firm and the pre-merger price. Consider first the effect of price coordination among the potentially merging firms, as it occurs after a merger has taken place. Under joint profit maximization, we need to add to the FOC in (3.A.14) the effect of a change in $\hat{p}_j$ on the profits of the other potentially merging firms. This effect is given by the following expression:[8]

$$\sum_{i\neq j} \left.\frac{\partial \hat{\pi}_i}{\partial \hat{p}_j}\right|_{\hat{p}_j=\hat{p}^*} = \hat{p}^* \left[\frac{1}{k}\frac{1-\bar{x}^k}{1-\bar{x}} - (\bar{x}-\tilde{p}^*+\hat{p}^*)^{k-1} + (k-1)\int_0^{\bar{x}-\tilde{p}^*} (\varepsilon+\hat{p})^{k-2}(\varepsilon+\tilde{p}^*)^{n-k}\, d\varepsilon\right] > 0 \tag{3.A.15}$$

We note now that the LHS of (3.A.14) decreases in $\hat{p}^*$. This is because its derivative is equal to

$$\begin{aligned}
& -(\bar{x}-\tilde{p}^*+\hat{p}^*)^{k-1} + (k-1)\int_0^{\bar{x}-\tilde{p}^*} (\varepsilon+\tilde{p}^*)^{n-k}(\varepsilon+\hat{p}^*)^{k-2}\, d\varepsilon \\
& < -(\bar{x}-\tilde{p}^*+\hat{p}^*)^{k-1} + (k-1)\,\bar{x}^{n-k}\int_0^{\bar{x}-\tilde{p}^*} (\varepsilon+\hat{p}^*)^{k-2}\, d\varepsilon \\
& = -(\bar{x}-\tilde{p}^*+\hat{p}^*)^{k-1}\left(1-\bar{x}^{n-k}\right) - \bar{x}^{n-k}\hat{p}^{*k-1} < 0
\end{aligned}$$

Since (3.A.15) is positive, this implies that the internalization-of-pricing-externalities effect shift upwards the joint reaction function $\nu_0(\hat{p})$ of the potentially merging stores. Consequently, since the reaction function of the non-merging stores is upward sloping (due to strategic complementarity of the price-strategies) the internalization-of-pricing-externalities effect can only increase further the price of the non-merging stores, as expected.

To conclude, we show that in this modified model once the merging firms coordinate their prices their payoff is independent of whether the keep the k stores or shut down all but one.[9] To see this, multiply the sum of (3.A.14) and (3.A.15) by k to obtain:

$$\begin{aligned}
& \frac{1-\bar{x}^k}{1-\bar{x}}(1-\bar{x}-\hat{p}^*) + \left(\bar{x}^k - (\bar{x}-\tilde{p}^*+\hat{p}^*)^k\right) + k\int_0^{\bar{x}-\tilde{p}^*} (\varepsilon+\tilde{p}^*)^{n-k}(\varepsilon+\hat{p}^*)^{k-1}\, d\varepsilon \\
& + \hat{p}^*\left[\frac{1-\bar{x}^k}{1-\bar{x}} - k(\bar{x}-\tilde{p}^*+\hat{p}^*)^{k-1} + k(k-1)\int_0^{\bar{x}-\tilde{p}^*} (\varepsilon+\hat{p})^{k-2}(\varepsilon+\tilde{p}^*)^{n-k}\, d\varepsilon\right] \\
& = 1 - (\bar{x}-\tilde{p}^*+\hat{p}^*)^{k-1}(\bar{x}-\tilde{p}^*+(k+1)\hat{p}^*) \\
& + k\int_0^{\bar{x}-\tilde{p}^*} (\varepsilon+\tilde{p}^*)^{n-k}(\varepsilon+\hat{p}^*)^{k-2}(\varepsilon+k\hat{p}^*)\, d\varepsilon
\end{aligned}$$

[8] Details on how to obtain this formula, which are available from the authors upon request, are omitted to save space.

[9] Of course, in our model, consumer search behavior is only consistent with equilibrium pricing if the merged entity sell all products at a single point-of-sale.

This last equation is exactly identical to the LHS of our FOC (3.11). These arguments together imply that $\tilde{p}^* > p^*$; hence, $\frac{\hat{\pi}_i}{k} - \pi^* > 0$.

(ii) We now prove the second statement. For this we need to show that $\hat{\pi}^*/k - \tilde{\pi}^* > 0$. It has been shown in the proof of Proposition 3.1 that when search cost is large $\hat{p}^* \to p_k^m$ and $\tilde{p}^* \to \frac{1}{2}$. Therefore, for the difference between the post-merger payoff of a merging firm $\hat{\pi}_i^*/k$ and the post-merger payoff of a non-merging firm, $\tilde{\pi}^*$, we have:

$$\begin{aligned}
\lim_{\bar{x}\to 1/2} \frac{\hat{\pi}_i^*}{k} - \tilde{\pi}^* = \lim_{\bar{x}\to 1/2} \Bigg[& \frac{\hat{p}^*}{k} \Big(1 - (\bar{x} - \tilde{p}^* + \hat{p}^*)^k \\
& + k \int_0^{\bar{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-1} \, d\varepsilon \Big) \\
& - \tilde{p}^* \left(\frac{(\bar{x} - \tilde{p}^* + \hat{p}^*)^k}{n-k} \left(1 - \bar{x}^{n-k}\right) \right. \\
& \left. + \int_0^{\bar{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon \right) \Bigg] \\
& = \frac{p_k^m}{k} \left(1 - (p_k^m)^k\right) - \frac{1}{2} \frac{(p_k^m)^k}{n-k} \left(1 - 2^{k-n}\right) \\
& = \frac{1}{k} p_k^m \left(1 - (p_k^m)^k\right) - \frac{1}{2\,(n-k)} (p_k^m)^k \left(1 - \frac{1}{2^{n-k}}\right) \\
& = \frac{1}{k} p_k^m \left(1 - (p_k^m)^k\right) - \frac{(p_k^m)^k}{2} \int_{1/2}^1 \varepsilon^{n-k-1} d\varepsilon
\end{aligned}$$

This expression is increasing in n because its derivative with respect to n equals

$$-\frac{(p_k^m)^k}{2} \int_{1/2}^1 \varepsilon^{n-k-1} \ln \varepsilon d\varepsilon > 0$$

Then

$$\begin{aligned}
& \lim_{\bar{x}\to 1/2} \frac{\hat{\pi}_i^*}{k} - \tilde{\pi}^* \geq \frac{p_k^m}{k} \left(1 - (p_k^m)^k\right) - \frac{1}{2} \frac{(p_k^m)^k}{k+1-k} \left(1 - 2^{k-k-1}\right) \\
= & \frac{p_k^m}{k+1} - \frac{(p_k^m)^k}{2} \left(1 - \frac{1}{2}\right) = \frac{p_k^m}{k+1} - \frac{1}{4\,(k+1)} = \frac{1}{4\,(k+1)} \left[4p_k^m - 1\right] > 0
\end{aligned}$$

where the first inequality follows form replacing n by $k+1$. ■

Proof of Proposition 3.3. (i) We first note that the equilibrium of Proposition 3.1 has $\hat{p}^* > \tilde{p}^*$ when the search cost is sufficiently high. The difference between post-and pre-merger total industry profits, denoted as $\Delta\Pi$, is:

$$\Delta\Pi = \hat{\pi} + (n-k)\tilde{\pi} - n\pi^*$$

Using the expressions for profits above, we have

$$\begin{aligned}\Delta\Pi = {} & \hat{p}^* \left(1 - (\overline{x} - \tilde{p}^* + \hat{p}^*)^k + k \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-1} d\varepsilon\right) \\ & + \tilde{p}^* \left((\overline{x} - \tilde{p}^* + \hat{p}^*)^k \left(1 - \overline{x}^{n-k}\right) + (n-k) \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k d\varepsilon\right) \\ & - p^* (1 - p^{*n})\end{aligned}$$

Note now that this expression is clearly increasing in $\hat{p}^*$ (the derivative of the first line, by the FOC, is zero and that of the second line is positive). Hence,

$$\Delta\Pi > \left.\Delta\Pi\right|_{\hat{p}^*=\tilde{p}^*} = \tilde{p}^* (1 - \tilde{p}^{*n}) - p^* (1 - p^{*n}) \tag{3.A.16}$$

Observe next that (3.A.16) increases in $\tilde{p}^*$ because its derivative with respect to $\tilde{p}^*$ equals $1 - (n+1)\,\tilde{p}^{*n}$. Therefore,

$$\tilde{p}^* (1 - \tilde{p}^{*n}) - p^* (1 - p^{*n}) > p^* (1 - p^{*n}) - p^* (1 - p^{*n}) = 0$$

where the inequality follows from using the result in the proof of Proposition 3.2 that $\tilde{p}^* > p^*$ and replacing $\tilde{p}^*$ by p^*.

(ii) In the pre-merger market, consumer surplus is given by

$$\begin{aligned}CS^{pre-merger} = {} & \frac{1-\overline{x}^n}{1-\overline{x}} \int_{\overline{x}}^1 (\varepsilon - p^*)\, d\varepsilon - s \frac{1-(n+1)\,\overline{x}^n + n\overline{x}^{n+1}}{1-\overline{x}} \\ & + n \int_{p^*}^{\overline{x}} \varepsilon^{n-1} (\varepsilon - p^* - ns)\, d\varepsilon - nsp^{*n}\end{aligned}$$

In the post-merger market, consumer surplus is given by $CS^{post-merger} = \widehat{CS} + \widetilde{CS} + CS_{\varnothing}$, where the expressions for $\widehat{CS}$, $\widetilde{CS}$ and $CS_{\varnothing}$ are given in (3.13), (3.14) and (3.15).

When $s \to 1/8$, $\overline{x} \to 1/2$, $p^* \to 1/2$ and $\hat{p}^* \to p_k^m$. Then, we can establish the comparison

$$lim_{s\to 1/8} \left[CS^{post-merger} - CS^{pre-merger}\right] = \int_{p_k^m}^1 (\varepsilon - p_k^m)\, d\varepsilon^k - \frac{1}{8} > 0$$

The proof is now complete. ■

Chapter 4

Collusion and Search

4.1 Introduction

A firm is not a price-taker in an oligopolistic market. Its unilateral decisions to increase its output and(or) decrease its price negatively affect the prices and profits of its competitors. These negative pricing (quantity setting) externalities imply that sellers can be better off if they coordinate their decisions.

Unfortunately a collusive equilibrium cannot be sustained if market interaction lasts for only one period. This happens because a single firm always increases its profit by deviating from the collusive price and output, given that its competitors adhere to the joint profit maximization strategy. However, things are quite different if sellers interact for more than one period. Friedman (1971) has shown that a collusive equilibrium may be sustained if firms agree to revert endlessly to the non-cooperative Nash equilibrium after at least one deviation is observed, and the discount factor is sufficiently high. If the sellers agree on such a strategy then, before it departs from the collusive set up, a potential deviant needs to compare the short term deviation gain with the deviation loss. The deviation gain is the difference between the deviation and collusive profits of a firm. The deviation loss is the accumulated sum of the discounted decrease in future profits of a firm due to the implemented punishment. Consequently, a cartel is sustainable if the deviation gain is less than the deviation loss. On the contrary, if the deviation gain is above the deviation loss then a cartel cannot survive.

The degree of market transparency is acknowledged to have an impact on cartel stability. It has been discussed by Stigler (1964) and later analyzed in more detail by Green and Porter (1984), Abreu et al. (1985) and Ivaldi et al. (2003a) that collusion is harder to sustain if firms cannot observe the choices of their competitors easily. If there is a positive probability that the choice of a single firm is not observed by its rivals then there is a chance that the deviant does not get caught immediately after the first deviation. In such a case the firm would deviate the next period again. Thus, the deviation gain increases if the probability that the deviation is not observed increases. Furthermore, the deviation loss decreases if firms do not observe each other perfectly. This happens because the punishment phase is postponed, which implies that the punishment is less severe if market transparency decreases.

The collocation '*market transparency*' can be perceived in two ways. As mentioned in the previous paragraph, a market can be transparent from the firms' point of view. In that case the sellers observe each others actions without any hindrance. However, market transparency can also be seen from the consumers' point of view. A consumer needs to observe all the offers of firms if she wants to choose the option with the highest utility. However, sometimes the information about products is lacking or has to be found at some positive costs. Then a market is non-transparent from the perspective of consumers.

The effects of costly consumer search on market equilibrium have been broadly discussed in the economics literature. It has been proved in the models of Burdett and Judd (1983), Janssen and Moraga-González (2004), Stahl (1989) and Janssen et al. (2005) that firms charge high and low prices with a positive probability if consumer search is costly and firms sell homogeneous products. Additionally, Wolinsky (1986) and Anderson and Renault (1999) have demonstrated that if firms sell horizontally differentiated products and consumer search is costly then the competitive equilibrium price is higher than if consumer search costs are zero. However there have not been many attempts to analyse how the search costs affect the incentives to collude.

Nilson (1999) has introduced a fraction of zero search cost consumers in the model of Burdett and Judd (1983) and analysed the stability of a cartel in a duopoly market. He has found that the cartel is easier to sustain if the search cost decreases. This happens because the expected price decreases relatively fast if the search costs decrease. Therefore, the punishment becomes harder, which makes a cartel more stable. Meanwhile Schultz (2005, 2009) has analysed collusion in the Hotelling model. In his set-up firms observe each other easily. However, for some exogenous reason a fraction of consumers are not informed about the prices of both firms; the rest of the buyers are perfectly informed. The uninformed consumers form their expectations about the prices and can visit only one firm. Schultz (2005) finds that the gain from the deviation increases if the market becomes more transparent. However, the demand of a firm is more elastic in a competitive market if more consumers are informed. Thus, the punishment becomes stronger if the market is more transparent. The effect of market transparency on the deviation gain is weaker than on the punishment if products are sufficiently differentiated. Therefore, the interval of the discount factor values, where collusion is sustainable, gets narrower if the fraction of uniformed consumers decreases (the market becomes more transparent). He also shows that if products become almost homogeneous then "*there is almost no impact of market transparency on the scope for collusion*."

In this chapter we analyze how cartel stability is affected by costly consumer search. We follow the sequential consumer search set-up that was derived by Wolinsky (1986) and analyzed in a more general form by Anderson and Renault (1999). According to the model assumptions, sellers can perfectly observe each other. However, their customers need to search for the highest utilities at positive costs. The analysis reveals that both the deviation gain and the deviation loss are affected by the search cost. Some buyers do not check all the alternatives before they make their purchase decisions because of costly search. Then less potential customers observe the deviation price of a cheating seller. As a result, deviating from the collusive equilibrium becomes less attractive. The non-cooperative equilibrium price increases with the search cost. Therefore, the punishment is less severe in a costly search market than in a fully transparent market. The deviation gain is sufficiently more sensitive to the search cost than the deviation loss for the most of parameter values in our model. Therefore, collusion is more stable if the search cost is higher.

Our result is in line with the findings of Schultz (2005). However, both models differ in two aspects. First of all, the uninformed consumers check only one firm in the model of Schultz (2005), whereas the searching consumers may check any number of alternatives (including all) in our model. Secondly, we introduce a search cost that affects a consumer's consideration set and search behaviour, whereas the fraction of uninformed consumers is determined exogenously in the model of Schultz (2005). Thirdly, we note that distance related transportation costs cannot be interpreted as the search costs in a Hotelling model with horizontally differentiated products. Hence, there are no search costs in the model of Schultz (2005). Finally, consumers do not know both the actual match values and the prices before they visit firms in our model. On the contrary, the uninformed consumers know their locations on the line in the model of Schultz (2005). Hence, only actual prices are unknown to them.

Cartel stability depends on the degree of product differentiation. However, this effect depends on the model assumptions a lot. The market power of firms increases if product heterogeneity goes up. Hence, the deviation loss decreases with the degree of product differentiation. If products are differentiated then a deviant must set its price lower than in a homogeneous product market because consumers see significant differences between the alternatives. Therefore, the deviation gain often decreases with the degree of product differentiation. Hence, whether a cartel is more or less stable when the degree of product heterogeneity increases depends on the parameters of the model. It has been shown by Deneckere (1983), Albæk and Lambertini (1998) and Rothschild (1997) that collusion becomes more stable if products are sufficiently differentiated and become more differentiated and the market demand is linear in prices. However, if products are relatively close substitutes then an increase in the degree of product differentiation works the other way around: a cartel is less stable if products are more differentiated.

In Hotelling-type models cartel stability is monotonic in the degree of product differentiation. It has been shown by Chang (1991) and Ross (1992) that the deviation gain decreases with the degree of product differentiation. However, the deviation loss also decreases if products become more differentiated. Nevertheless, collusion is less stable if products become more differentiated. Additionally, Jehiel (1992) shows that collusion encourages firms to decrease the degree of product differentiation if the joint profit is shared not equally and there are no monetary transfers among the cartel members. Furthermore, Häckner (1995) has shown that colluding firms prefer to change the distance between each other on a Hotelling line if the discount factor varies. In his analysis two sellers maximize their joint profit with respect to the degree of product differentiation so that collusion is sustained for a fixed discount factor value. The firms move towards the ends of the line if the discount factor increases. This result holds if the sellers are relatively close to each other and the discount factor is relatively small. However, if the discount factor is relatively high already and its value goes up further then the sellers prefer to limit the degree of product differentiation.

We look at the effect of product differentiation in our model as well. We find that the relationship between cartel stability and the degree of product differentiation depends on model parameters. If the match value is distributed uniformly and the search costs approach zero then the degree of product differentiation does no have any effect on cartel stability. If search costs are positive and the match value is distributed uniformly then a cartel is less stable if products are more differentiated. However, if the match value is distributed according to an exponential family distribution, there are two firms in the market and products are almost homogeneous then a cartel is more stable if the degree of product heterogeneity increases.

We apply the method of Perloff and Salop (1985) to account for the degree of product heterogeneity, i.e. we multiply the match value with the preference intensity parameter in the utility function. Hence, one could expect that the effect of product heterogeneity on cartel stability must be somehow similar to the effect in a Hotelling-type model. However, it is not so. The difference arises because the market is not fully covered in our set-up and the market is typically fully covered in Hotelling models. Then, an increase in the preference intensity parameter has two effects in our model. Firstly, it increases the mean valuation of a product, which encourages the firms to increase their prices. Secondly, the variance of the match values increases, which implies more product differentiation. If products are more differentiated then consumers search more intensively and firms have incentives to lower their prices. Which effect is stronger depends on the parameters of the model. Hence, whether an increase in product heterogeneity leads to a more or less stable cartel also depends on the model parameters.

We describe the consumer search behaviour and the search rule in the subsequent section. We derive the competitive and joint profit maximizing prices and profits in section 4.3. The sustainability of a cartel with different search cost values is analysed in sections 4.4 and 4.5. We show how the degree of product differentiation affects collusion in section 4.6. Finally, some concluding remarks are laid down in section 4.7. More complicated proofs and derivations are presented in the appendix of the chapter.

4.2 Consumer behaviour

There is a mass of consumers on the demand side of the economy, and we normalize this mass to one. Consumers have different preferences for the product characteristics and each consumer buys one item at most. Heterogeneity in consumer tastes implies that the same variety is rated differently by distinct buyers.[1] In other words, if we denote the valuation of consumer i for item j by ε_{ij} then $\varepsilon_{ij} \neq \varepsilon_{mj}$, $\forall m \neq i$. We assume that the valuation (or *match value*) is distributed in the interval between

[1] The terms *a consumer*, *a shopper*, *a customer* and *a buyer* are used interchangeably as synonyms further on.

zero and $\bar{\varepsilon} > 0$ according to a continuous, twice differentiable function $F(\varepsilon)$ with a positive density $f(\varepsilon)$ and an increasing hazard rate. Additionally, we require that $f(y) + pf'(y) > 0,\ \forall p \leq y$. There is no correlation between the match values across consumers, products and time. The choices that a consumer made in the past have no effect on the present purchase decisions.[2]

A seller cannot identify how much one or another consumer likes its variety. Thus, price discrimination is not possible and all consumers pay the same price p_j if they buy a product from firm j. We assume that the total utility of consumer i, who buys product j, equals the difference between the match value ε_{ij} and price p_j

$$u_{ij} = \varepsilon_{ij} - p_j \tag{4.1}$$

A consumer prefers to buy the alternative that gives her the highest utility. However, she does not know what variety and at what price is sold in each shop exactly. The shopper knows the distribution of ε and has some expectations about prices. Though, she needs to go to every shop and check what is on the counter exactly before she decides to buy anything if she buys at all. The consumer can visit and check one shop at a time. She visits sellers one after another using an optimal sequential search strategy. Search is without replacement and with costless recall.

Every visit to a shop costs s for the consumer. The search cost has a negative impact on the buyer's utility. Thus, the consumer needs to find a balance between her wish to find the highest utility in the market and the search cost. According to Weitzman (1979), a rational consumer should search as long as the gain from one more search is greater than the search cost. If the search cost exceeds the expected additional utility from an extra search then the shopper should stop looking for other alternatives and choose the best among observed ones.

Let us assume that utility u_{ij} is the highest observed utility by consumer i. If the buyer considers going to firm $l \neq j$ then the gain from an additional search equals

$$\int_{u_{ij}}^{\bar{\varepsilon}-p_l} \left(u_{il} - u_{ij}\right) dF\left(u_{il}\right) \tag{4.2}$$

[2] The assumption that the purchase decisions are independent from the choices in the past is realistic in the markets where the assortment in the shops changes between the purchases of a consumer. For instance, clothing collections change every half year. New fashion trends imply changes in furniture, cosmetics, and construction materials. The producers of home appliances, computers, phones and cars update their models due to rapid technological progress. Thus, a buyer always finds different offers whenever she comes to buy a washing machine, a fridge or a digital camera.
The past purchase decisions and experience may imply that a consumer has higher (or lower) preferences over some brands. However, if there are new models every shopping season then the brand loyalty affects only the search order of a consumer but not the valuation of a particular product.

The visit to shop l costs s. Consumer i is indifferent between taking alternative j and searching in shop l if the following equality is satisfied

$$\int_{\varepsilon_{ij}-p_j+p_l}^{\bar{\varepsilon}} \left(\varepsilon_{il} - \left(\varepsilon_{ij} - p_j + p_l\right)\right) dF\left(\varepsilon_{il}\right) = s \tag{4.3}$$

Let us define a variable $\bar{x}$ such that $\bar{x} = \varepsilon_{ij} - p_j + p_l$ if (4.3) is satisfied. Then (4.3) may be rewritten as

$$\int_{\bar{x}}^{\bar{\varepsilon}} \left(\varepsilon_{il} - \bar{x}\right) dF\left(\varepsilon_{il}\right) = s \tag{4.4}$$

The variable $\bar{x}$ takes on a unique value in the interval between zero and $\bar{\varepsilon}$ for any fixed value of s and decreases with the search cost.

Very high search costs prohibit consumers from entering the market. Therefore, we restrict the maximum search cost in such a way that a consumer searches at least once even if firms set the price that maximizes the profit of the monopolist that sells n varieties. We denote this price by p^c.[3]

The maximum search cost $\bar{s}$ cannot be higher than the expected utility that a consumer gets after she pays p^c, i.e.

$$\bar{s} = \int_{p^c}^{\bar{\varepsilon}} \left(\varepsilon - p^c\right) dF(\varepsilon)$$

Thus, the minimum value of $\bar{x}$ cannot be less than p^c.

As said above, consumer i follows an optimal sequential search strategy. Thus, she terminates her search in firm j if $\bar{x} < \varepsilon_{ij} - p_j + p_l$ because the gain from an additional search is below the cost of the visit to firm l. However, if $\bar{x} > \varepsilon_{ij} - p_j + p_l$ then the shopper continues searching in firm l.

4.3 One period interaction

4.3.1 Non-cooperative equilibrium

There are $n \geq 2$ firms selling horizontally differentiated products in the market. Each firm supplies one variety at a common unit cost, which we normalize to zero.

[3] Later we show that the joint profit maximizing price of a cartel is the same as the price that maximizes the profit of the monopolist that sells n varieties. We use the superscript c to refer to the price and the profit of a cartel in the subsequent sections. Hence, we use the same notation for the price of the monopolist.

$p^c = \arg\max_p \; p \Pr\left[\max\left\{\varepsilon_j\right\}_{j=1,\ldots n} > p\right] = \arg\max_p \; p\left(1 - F(p)^n\right)$

Sellers are informed about the distribution of match values, consumers' expectations and the search rule. Thus, every seller needs to choose its profit maximizing price, which coincides with consumers' expectations in equilibrium.

Consider a firm j that contemplates charging price p_j. There is no vertical product differentiation. In addition, consumers know that each seller offers one variety and firms have the same production cost structure. As a result, buyers expect that all shops charge the same price p^*.[4]

Weitzman (1979) proved that rationally searching consumers should rank all the sellers according to their reservation utilities in a decreasing order and start searching from the top of the list. The *reservation utility* at firm l is the utility level that a buyer should observe in firm $j \neq l$ and to make her indifferent between searching in firm l and terminating her search in firm j. According to the search rule written in (4.4), a consumer stops searching in firm j and does not go to firm l if $\varepsilon_j \leq \bar{x} - p_l + p_j$. Then the lowest utility level in firm j (or the reservation utility at firm l) that prevents a buyer from searching in firm l is $\bar{x} - p_l$. Consumers expect that all sellers have the same price p^* in the competitive market. Thus, the reservation utilities at all firms are $\bar{x} - p^*$ and consumers visit shops in a random order.

Firm j may be visited in the first, second and any other position up to the n^{th} with the probability $1/n$. The consumer, who has reached firm j, observes price $p_j \neq p^*$. The buyer interprets price p_j as the deviation price of firm j only. Therefore, she expects to see price p^* in other shops. As a result, the reservation utilities at not-yet-visited firms are unaffected by the deviation price of firm j, and the consumer terminates her search in firm j if ε_j is greater than or equal to $\bar{x} - p^* + p_j$. Otherwise, the shopper goes to the next firm. Since match values are distributed in the interval between zero and $\bar{\varepsilon}$, the probability that the consumer, who has arrived at firm j, terminates her search here equals $1 - F\left(\bar{x} - p^* + p_j\right)$.

All shops, except shop j, choose equilibrium price p^*. Therefore, the probability that a consumer has arrived at firm j equals the probability that the match values have been less than or equal to $\bar{x}$ in the firms that have been visited before firm j. The number of consumers is normalized to one. Therefore, the probability that a single consumer has arrived at firm j and terminated her search there equals *"the fresh demand"* of firm j. We denote this demand by f_j.

$$f_j = \frac{1}{n}\sum_{i=1}^{n} F\left(\bar{x}\right)^{i-1}\left(1 - F\left(\bar{x} - p^* + p_j\right)\right) = \frac{1}{n}\frac{1 - F\left(\bar{x}\right)^n}{1 - F\left(\bar{x}\right)}\left(1 - F\left(\bar{x} - p^* + p_j\right)\right)$$

[4] The terms *a firm*, *a shop*, and *a seller* are used interchangeably, unless it is stated differently.

The utility in firm j is less than $\bar{x} - p^*$ for some consumers. These buyers do not stop in the firm and continue searching further. However, several consumers visit all sellers in the market and decide to return to firm j and buy there. This happens if the utility in firm j is the highest in the market and is greater than zero. If the highest observed utility is negative then a consumer decides not to buy anything. The number of consumers, who return to firm j and buy there, are called *"the returning demand"* of firm j and it is labeled by r_j. The returning demand of firm j equals the probability that a consumer has not terminated her search in it and in any other shop but has returned and has made a purchase in firm j.

$$\begin{aligned} r_j &= \Pr\left[0 \leq \varepsilon_j - p_j \leq \bar{x} - p^* \text{ and } \varepsilon_j - p_j > \max\left\{\varepsilon_l - p^*\right\}_{\forall l \neq j}\right] \\ &= \int_0^{\bar{x}-p^*} F\left(\varepsilon + p^*\right)^{n-1} f\left(\varepsilon + p_j\right) d\varepsilon \end{aligned}$$

The profit function of firm j equals the total income of firm j because its constant unit production cost is normalized to zero. We denote this function by π_j.

$$\pi_j = p_j\left(f_j + r_j\right) \tag{4.5}$$

Firm j chooses such p_j that makes its first order condition equal to 0. This choice guarantees the maximum profit of firm j because π_j is concave in p_j[5]. The seller finds it optimal to set p^* in equilibrium and the first order condition of firm j simplifies to equation (4.6).

$$1 - \frac{1 - F\left(\bar{x}\right)^n}{1 - F\left(\bar{x}\right)} p^* f\left(\bar{x}\right) - F\left(p^*\right)^n + n p^* \int_{p^*}^{\bar{x}} F\left(\varepsilon\right)^{n-1} f'\left(\varepsilon\right) d\varepsilon = 0 \tag{4.6}$$

There is a unique value of p^* that satisfies (4.6). This can be shown by taking the

[5] See the appendix for expression of the second order derivative.
We have assumed that firm j has deviated to a relatively smaller or higher price than p^* so far. However, it is not profitable for the seller to deviate to a much higher or to a much lower price than p^*. It is not profitable for the firm to deviate to any much smaller price than p^* because its profit function increases with any deviation price which is smaller than p^*. Furthermore, it is not profitable for the seller to deviate to a very high price such that no consumer stops in its shop. If the firm deviates to such a high price then its profit equals

$$\pi_j = p_j \int_{p_j}^{\bar{\varepsilon}} F\left(\varepsilon - p_j + p^*\right)^{n-1} f(\varepsilon) d\varepsilon$$

Then the first order condition of the firm at point p^* is negative because

$$\left.\frac{\partial \pi_j}{\partial p_j}\right|_{p_j = p^*} = \int_0^{\bar{\varepsilon}-p^*} F\left(\varepsilon + p^*\right)^{n-1} \left(f\left(\varepsilon + p^*\right) + p^* f'\left(\varepsilon + p^*\right)\right) d\varepsilon - p^* f\left(\bar{\varepsilon}\right) < 0 \quad \text{if } \bar{x} < \bar{\varepsilon}$$

derivative of the LHS of (4.6) with respect to p^* and checking the sign of the LHS of (4.6) for the highest and lowest possible values of p^*. It is shown in the appendix that the LHS of (4.6) decreases with p^*. It is not difficult to observe that the LHS of (4.6) equals one if $p^* = 0$ and equals $\frac{1-F(\bar{x})^n}{1-F(\bar{x})}\left(1 - F\left(\bar{x}\right) - \bar{x} f\left(\bar{x}\right)\right) < 0$ if $p^* \to \bar{x}$. Furthermore the competitive equilibrium price increases with the search cost.[6]

A high search cost corresponds with a low threshold value for ε. Consumers are less picky if the search cost is high. This implies that consumers search less intensively and compare less alternatives if the value of $\bar{x}$ goes down. As a result, firms can charge higher prices if s increases. On the contrary, if the search cost decreases then consumers compare more offers and, consequently, firms compete for their customers more fiercely. Then the equilibrium price is pushed down.

After some arithmetic manipulations we get that the equilibrium profit of a firm, π^*, can be written as follows

$$\pi^* = \frac{p^*}{n}\left(1 - F\left(p^*\right)^n\right) \tag{4.7}$$

The partial derivative of π^* with respect to p^* is positive because $p^* < p^c$.[7] This implies that the equilibrium profit of a firm increases with the search cost.

4.3.2 Joint price setting

The profit of a firm increases if the equilibrium price goes up. Horizontally differentiated products selling firms exert pricing externalities on each other. Hence, the equilibrium price and profits increase if firms maximize their joint profit instead of competing with each other. The sellers are symmetric in our model. Thus, the joint profit maximization problem is identical to the profit maximization problem of the monopolist that owns n single-variety shops. Consumers expect to see p^c in every shop in collusive equilibrium, where the superscript c refers to collusion.[8] Further-

[6] We show in the appendix that the partial derivatives of the LHS of (4.6) with respect to p^* and with respect to $\bar{x}$ are negative. Then by the Implicit function theorem we get that p^* increases with s.

[7]
$$\frac{\partial \pi^*}{\partial p^*} = \frac{1}{n}\left(1 - F\left(p^*\right)^n - nF\left(p^*\right)^{n-1} p^* f(p^*)\right) > 0$$
The inequality follows from the fact that the competitive price is less than the joint profit maximizing price.

[8] The new expected price differs from p^*. Colluding firms maximize their joint profit and the expectations of consumers must match with the profit maximizing choice of a cartel in equilibrium. We do not analyse how the expectations of consumers move from p^* towards p^c. On the one hand, it could be a common knowledge that firms collude; however, the competition authorities cannot prove the existence of a cartel. On the other hand, consumers may update their expectations because of their experience in transitional periods. Furthermore, the joint payoff function does not depend on the expected price.

more, the buyers expect that all firms charge the same price. So they visit the sellers a random order because the reservation utilities at all shops are the same.

The sellers consider the joint profit maximizing price p, by taking p^c into account. As well as in a pre-merger market, the demand of every shop consists of its fresh and returning demands. The reservation utility at every shop equals $\bar{x} - p^c$. The joint profit equals the sum of the pay-offs of all firms and can be written as follows.

$$\begin{aligned}\Pi^c &= \left(\frac{1 - F\left(\bar{x} - p^c + p\right)^n}{1 - F\left(\bar{x} - p^c + p\right)} \left(1 - F\left(\bar{x} - p^c + p\right)\right) + n \int_p^{\bar{x} - p^c + p} F\left(\varepsilon\right)^{n-1} f\left(\varepsilon\right) d\varepsilon \right) \\ &= p\left(1 - F\left(p\right)^n\right)\end{aligned}$$

The expectations of consumers coincide with the choice of the monopolist. In other words, $p = p^c$ in one period collusive equilibrium. As a result, we set $p = p^c$ in the first order condition of the monopolist and it simplifies to (4.8).

$$\left.\frac{\partial \Pi^c}{\partial p}\right|_{p=p^c} = 1 - F\left(p^c\right)^n - p^c n F\left(p^c\right)^{n-1} f\left(p^c\right) = 0 \tag{4.8}$$

The joint profit maximizing price p^c is unique because the LHS of (4.8) decreases with p^c, it equals zero if $p^c = 0$ and is negative if $p^c = \bar{\varepsilon}$. This price does not depend on the search cost and increases with n.[9] Every firm gets the same share of the joint profit. Thus, the profit of one coalition partner equals the ratio between the joint profit and n. We denote this profit by π^c.

$$\pi^c = \frac{\Pi^c}{n} = \frac{p^c}{n}\left(1 - F\left(p^c\right)^n\right) \tag{4.9}$$

4.4 Collusion

The joint profit maximizing condition can be written in the following way:

$$\left.\frac{\partial \Pi^c}{\partial p_j}\right|_{p_j=p^c} = \left.\frac{\partial \pi_j}{\partial p_j}\right|_{p_j=p^c} + \sum_{i \neq j} \left.\frac{\partial \pi_i}{\partial p_j}\right|_{p_j=p^c} = 0 \tag{4.10}$$

The profit of firm j is maximized when the first term on the LHS of (4.10) equals

However, the expected price has an impact on the reservation utility, which implies that the search intensity depends on the expected price. Consumers search more if the expected price is smaller than p^c and search less if the expected price is more than p^c. Nevertheless, it does not have any effect on the joint profit maximizing price p^c.

[9] See the appendix for a proof.

zero. Firms sell substitutes in a horizontally differentiated product market. Thus, the second term on the LHS of (4.10) is not zero. Hence, p^c is not the own profit maximizing choice of firm j if other firms charge the joint profit maximizing price. Then a single seller prefers to deviate from the cartel, and the collusive equilibrium cannot be sustained if the game lasts only one period. However, deviating is not necessarily profitable from a long term perspective. If all sellers interact in the same market for several periods then fooled firms may punish the deviant in the period following the deviation. If the punishment is sufficiently hard then firm j may give up its plans about deviating with the purpose to avoid the punishment later.

We assume that firms interact for an infinite number of periods. Every period consumers search the sellers sequentially as it has been described in section 4.2 and make their choices independently from the choices in the past. The firms maximize the sum of their discounted profits. The discount factor δ is constant over time and is less than one. The sum of discounted profits is the highest if the firms maximize their joint profit every period. Therefore, there is a motivation for the sellers to start colluding.

Unilateral deviations from the cartel set-up must be prevented in order to make collusion sustainable. We consider that the sellers apply a grim trigger strategy in our model. To put differently, the firms set the joint profit maximizing price p^c every period if all the coalition members do this. If at least one shop deviates from the collusive price then all the sellers revert to the one period Nash equilibrium price for the rest of the time. Both the consumers and the firms observe the choice of every seller in the end of each period. Therefore, the punishment mechanism starts working the next period after the deviation is observed.

Consumers search because they do not know actual prices and match values in our model. Hence, the assumption that both the firms and all the consumers get informed about the deviation after the first deviation period may appear unreasonable at first sight. The expressions of profits and the condition of cartel stability simplifies a lot if this assumption is used. Hence, the comparative statics becomes easier. Moreover, we check what happens if not all consumers observe the deviation immediately in section 4.5. More particularly, there we assume that only the consumers, who visit the deviant, get informed about the deviation. We show that the qualitative results of the cartel stability analysis are the same in both cases.

A potential deviant compares the gain and the loss from the deviation. If the gain is higher than the loss then the seller deviates. On the contrary, if the loss exceeds the gain then the firm sticks to the cartel agreement. We denote the deviation profit by π^d. Then the deviation gain equals $\pi^d - \pi^c$. After a deviation has taken place, firms set p^* in the subsequent periods. The reversion to the one period Nash equilibrium price decreases one period profit of a firm by $\pi^c - \pi^*$. Thus, the profit of the deviant decreases by $\pi^c - \pi^*$ every period after the deviation. Hence, if we sum the discounted difference $\pi^c - \pi^*$ over all punishment periods then we get the total decrease in the profits of the firm - the deviation loss.

We start the comparison of the deviation gain and the deviation loss with the derivation of the deviation price. Afterwards we derive the expression of the deviation profit and the expressions of the deviation gain and the deviation loss. Finally, we look for the discount factor values that preserve cartel stability for different s values.

Deviation price. A single unilateral deviation does not affect the expectations of consumers about the prices of other firms. Thus, the buyers expect that other sellers charge p^c if they see p^d in one shop. As a consequence, the derivation of the payoff function of the deviant is similar to the derivation of the pay-off function of firm j in section 4.3.1. If we replace p^* with p^c and p_j with p^d in the pay-off function of firm j then we get the pay-off function of a deviating firm. Thus, the first order condition of the deviant may written as

$$\begin{aligned} &\frac{1-F(\bar{x})^n}{1-F(\bar{x})}\left(1-F\left(\bar{x}-p^c+p^d\right)-p^d f\left(\bar{x}-p^c+p^d\right)\right) \\ &+n\int_0^{\bar{x}-p^c} F(\varepsilon+p^c)^{n-1}\left(f\left(\varepsilon+p^d\right)+p^d f'\left(\varepsilon+p^d\right)\right)d\varepsilon = 0 \qquad (4.11) \end{aligned}$$

Claim 4.1. *There is a deviation price p^d that satisfies (4.11), and $p^* \leq p^d < p^c$.*

The deviation price varies with s. The higher the search cost is, the less consumers observe the deviation price. Therefore, a deviant does not attract many customers by lowering its price if the search cost is relatively high. Hence, there are less incentives to deviate from the cartel agreement if s is high. If the search cost decreases then the deviant can attract more customers by lowering p^d because more buyers observe the deviation price. However, the firm has incentives to raise its price if the number of its customers increases. The total effect of the search cost on the deviation price depends on whether the positive of the negative effect is stronger.

Claim 4.2. *The deviation price p^d increases with the search cost.*

There is an indirect negative effect of s on the quantity of the deviant that comes via the deviation price. A firm sells less if its price increases and the deviation price increases with s. However, there is a direct effect of s on the quantity of the deviant too. Consumers search less if the search cost goes up. Hence, the direct effect of the search cost on the quantity of the deviant is also negative. Then the quantity of the deviant decreases with the search costs. The negative search cost effect on the quantity is stronger than the effect on the deviation price. Therefore, the deviation profit decreases with s.

Claim 4.3. *The deviation profit π^d decreases with s.*

Cartel stability. The deviation is observed by all the firms in the end of the deviation period. Hence, it is followed by the punishment from the subsequent period for the rest of the time. A firm can earn the deviation profit only in one period and it deviates the first collusive period if it deviates at all.[10] A potential deviant compares the deviation gain with the deviation loss before it sets p^d in its shop. If the gain is higher than the loss then every firm deviates in the first collusive period and a cartel is not sustainable. However, if the deviation loss is higher than the deviation gain then the collusive agreement survives, as no firm wants to deviate from it. This cartel stability condition is written in (4.12).

$$\pi^d - \pi^c \leq \delta \left(\pi^c - \pi^*\right) + \delta^2 \left(\pi^c - \pi^*\right) + ... = \frac{\delta}{1-\delta} \left(\pi^c - \pi^*\right) \tag{4.12}$$

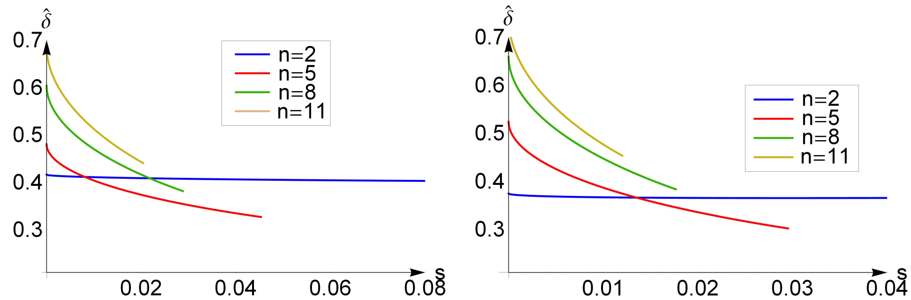
The factor $\delta / \left(1 - \delta\right)$ increases with δ. Therefore, there is some $\widehat{\delta}$ such that (4.12) becomes an equation if $\delta = \widehat{\delta}$. If $\delta > \widehat{\delta}$ then cartel members follow a collusive agreement; if the discount factor is smaller than the critical value then the deviation loss is less than the deviation gain and a cartel collapses in the first period.

Proposition 4.1. *Assume that $\varepsilon \sim U\left(0,1\right)$. Then the critical discount factor $\widehat{\delta}$ decreases with the search cost.*

This result implies that collusion is less sustainable if market transparency increases. The proof of the proposition is written in the appendix and here we provide only an intuitive explanation. The collusive profit does not depend on the search cost. Thus, only π^d and π^* change when s varies. It has been shown that the deviation profit decreases with the search cost. Hence, the gain from the deviation gets smaller if s goes up. The competitive profit rises if consumer search becomes more

[10] The discount factor δ is less than one, and the discounted deviation profit of a firm if it deviates in period t is $\delta^{t-1}\pi^d < \pi^d$. Therefore, the seller deviates in period one if it deviates at all.

costly. Thus, the deviation loss gets less severe if the search cost increases. Both sides of (4.12) decrease with s. However, the critical discount factor declines with the search cost because the negative effect of s on the deviation gain is stronger than the effect on the deviation loss (see Figure 4.1a).



(a) $\varepsilon \sim U\,(0, 1)$ (b) $F\,(\varepsilon) = \frac{e^{\varepsilon}-1}{e-1}$

Figure 4.1. The changes of $\widehat{\delta}$ with the search cost for different n values

We check whether the result of the proposition holds for other match value distributions. We assume that ε is distributed according to an exponential-type truncated distribution in the interval between zero and one. More particularly, we assume that $F\,(\varepsilon) = \frac{\exp\{\varepsilon\}-1}{e-1}$. This distribution function has an increasing hazard rate and satisfies $f\,(y) + pf'\,(y) > 0$, $\forall p \leq y$. The analytical expressions of the payoff functions and the critical discount factor are complicated if the exponential-type distribution is taken. Therefore, we present the simulation results only. For all $n \geq 3$ the critical discount factor decreases with s, given the new distribution of ε (see Figure 4.1b). However, if $n = 2$ then the value of $\widehat{\delta}$ varies differently with s. If there are two firm in the market then the critical discount factor decreases with s for the low search cost values and increases with s if the search cost is high. The deviant is searched by relatively many customers if there are a few firms in the market. Therefore, the quantity of the deviant decreases with s slower if there are less sellers. Furthermore, the exponential-type distribution function is convex and the probability that a customer has a very high valuation of a product is relatively high. Hence, consumers search less willingly with the exponential-type distribution of ε than with the uniform distribution. As a result, the positive effect of the search cost on the deviation price is weaker if ε is distributed exponentially. Thus, it happens that the deviation gain decreases with s slower than the deviation loss and $\widehat{\delta}$ increases with the search cost.

4.5 Transitional punishment period

It has been assumed that all the consumers were informed about the market price(s) at the end of each period in section 4.4. Therefore, firms set p^* in the first punishment period. However, it is not optimal for the sellers to set p^* in the first punishment period if some consumers do not observe the deviation in the deviation period and expect that firms collude in the next period. In this section we assume that only the fraction of consumers $\nu < 1$ observe the deviation price in the deviation period. Then ν consumers expect to see the punishment price in period $t+1$ if the deviation occurred in period t, whereas $1-\nu$ consumers have no idea about the deviation and expect to see p^c in the shops. The fraction of consumers, who observed the deviation price p^d in the deviation period, is endogenous. In fact, it equals the share of the buyers, who arrived at the deviating firm during the deviation period. Thus, $\nu = \frac{1}{n}\frac{1-F(\bar{x})^n}{1-F(\bar{x})}$.

On the contrary, all firms detect the deviation immediately and start the punishment in the following period. The competitive price p^* is not an optimal choice for the sellers in the first punishment period because there are two groups of consumers who expect two different prices in the market. Therefore, the firms charge a price $\widetilde{p} \neq p^*$ in the first punishment period. After all consumers observe $\widetilde{p}$, they recognize that the cartel broke down and the firms will compete for the rest of the time. Therefore, all the buyers expect to see p^* in period $t+2$ if the price $\widetilde{p}$ is set in period $t+1$. Hence, the firms charge $\widetilde{p}$ in the first punishment period and later set p^* for the rest of the time.

The first period punishment price is less than the collusive price because firms compete with each other. However, it is higher than the competitive price p^*. To show this we derive the payoff function of firm j which considers some deviation price $\widetilde{p}_j \neq \widetilde{p}$. The seller knows that there are two expected prices in the market. The fraction ν of consumers expect to see the punishment price $\widetilde{p}$, whereas, $1-\nu$ consumers wait for p^c. As a result, we can split the demand derivation of firm j in two parts. The first part of the demand is the demand from the informed consumers and the second part of the demand is from $1-\nu$ uninformed consumers.

All shops look alike for the informed consumers. The buyers sample firms randomly, because the reservation utilities at all sellers are the same $\bar{x} - \widetilde{p}$. Therefore, the demand from ν consumers is identical to the demand of firm j that has been derived in section 4.3.1. We only need to replace p^* with $\widetilde{p}$ and p_j with $\widetilde{p}_j$.

The fraction $1-\nu$ of consumers expect that all firms set p^c in their shops and get surprised by $\widetilde{p}_j$ in the shop of firm j. The buyers interpret the deviation as a mistake and do not change their expectations about not-yet-visited shops. Therefore, an uninformed consumer searches beyond firm j if $\varepsilon_j < \bar{x} - p^c + \widetilde{p}_j$. Other sellers, which the consumer has visited before firm j, have been left because their match values have been less than $\bar{x} - p^c + \widetilde{p}$. Thus, the fresh demand of firm j from the uninformed consumers equals the probability that an uninformed consumer has arrived at firm j and terminated her search here regardless of the position that firm j has been sampled:

$$f_j^1 = \frac{1}{n} \frac{1 - F\left(\bar{x} - p^c + \widetilde{p}\right)^n}{1 - F\left(\bar{x} - p^c + \widetilde{p}\right)} \left(1 - F\left(\bar{x} - p^c + \widetilde{p}_j\right)\right)$$

Some of the uninformed consumers do not terminate their search in any shop. However, they may return to firm j and buy here if the utility from firm j is positive and it is the highest in the market. Thus, we obtain the returning demand of firm j from the uninformed consumers which equals

$$\begin{aligned} r_j^1 &= \Pr\left[\max\left\{\varepsilon_l - \widetilde{p}; 0\right\}_{l=1,\ldots n-1} \leq \varepsilon_j - \widetilde{p}_j \leq \bar{x} - p^c\right] \\ &= \int_0^{\bar{x}-p^c} F\left(\varepsilon + \widetilde{p}\right)^{n-1} f\left(\varepsilon + \widetilde{p}_j\right) d\varepsilon \end{aligned}$$

Then the total payoff of firm j equals the sum of both demands times its price $\widetilde{p}_j$. We label this payoff $\widetilde{\pi}_j$.

$$\begin{aligned} \widetilde{\pi}_j = \frac{\widetilde{p}_i \nu}{n} &\left[\frac{1 - F\left(\bar{x}\right)^n}{1 - F\left(\bar{x}\right)} \left(1 - F\left(\bar{x} - \widetilde{p} + \widetilde{p}_j\right)\right) + n \int_0^{\bar{x}-\widetilde{p}} F\left(\varepsilon + \widetilde{p}\right)^{n-1} f\left(\varepsilon + \widetilde{p}_j\right) d\varepsilon\right] \\ &+ \widetilde{p}_j \left(1 - \nu\right) \left[f_j^1 + r_j^1\right] \end{aligned}$$

The choice of firm j coincides with the expectation of informed consumers in equilibrium. Therefore, the first order condition of the firm becomes

$$\begin{aligned} &1 - F\left(\widetilde{p}\right)^n - \nu \left[\frac{1 - F\left(\bar{x}\right)^n}{1 - F\left(\bar{x}\right)} \widetilde{p} f\left(\bar{x}\right) - n\widetilde{p} \int_0^{\bar{x}-\widetilde{p}} F\left(\varepsilon + \widetilde{p}\right)^{n-1} f'\left(\varepsilon + \widetilde{p}\right) d\varepsilon\right] \\ &- \left(1 - \nu\right) \left[\frac{1 - F\left(\bar{x} - p^c + \widetilde{p}\right)^n}{1 - F\left(\bar{x} - p^c + \widetilde{p}\right)} \widetilde{p} f\left(\bar{x} - p^c + \widetilde{p}\right)\right. \\ &\left. - n\widetilde{p} \int_0^{\bar{x}-p^c} F\left(\varepsilon + \widetilde{p}\right)^{n-1} f'\left(\varepsilon + \widetilde{p}\right) d\varepsilon\right] = 0 \end{aligned} \tag{4.13}$$

The LHS of (4.13) decreases with $\widetilde{p}$.[11] However, the derivative of the LHS with respect to p^c is positive.[12] Therefore, the derivative of $\widetilde{p}$ with respect to p^c is positive by the Implicit function theorem. It is not difficult to see that $\widetilde{p} = p^*$ if $p^c = p^*$. However, the collusive price is above p^*. Thus, $\widetilde{p} > p^*$. In addition, $\widetilde{p}$ is less than the monopoly price p^m that is set by a single variety monopolist.[13] To show this we set $\widetilde{p} = p^m$ and $p^c = \bar{x}$. Then we get that the LHS of (4.13) is negative,[14] which implies that $\widetilde{p} \leq p^m$.

The first period after deviating equilibrium profit of a firm equals

$$\widetilde{\pi} = \frac{\widetilde{p}}{n}\left(1 - F\left(\widetilde{p}\right)^n\right).$$

The profit is higher than π^* but less than π^c and we show that it increases with s. The first punishment period profit is affected by the size of the search cost via two sources. First of all, consumers search less in the first punishment period if the search cost increases. Therefore, an increase in s in the first punishment period has a positive effect on $\widetilde{p}$. This can be shown more formally by fixing the value of ν and taking the derivative of the LHS of (4.13) with respect to $\bar{x}$. The derivative is the sum of two derivatives that are similar to the derivative of (4.6) with respect to $\bar{x}$ and to the derivative of (4.11) with respect to $\bar{x}$. Therefore, the derivative of the LHS of (4.13) with respect to $\bar{x}$ for any fixed ν is negative and the effect of the search cost on $\widetilde{p}$ in the first punishment period is positive. Furthermore, there is the effect of s on $\widetilde{p}$ from the deviation period. The consumers, who did not observe the deviation price p^d in the deviation period, are less choosy than the consumers that visited the deviant because $\bar{x} - p^c + \widetilde{p} < \bar{x}$. Therefore, the firms may raise their prices if the fraction of $1-\nu$ consumers increases. The value of ν depends on the search cost and ν increases with $\bar{x}$. Therefore, the effect of s on $\widetilde{p}$ from the deviation period is positive. This fact may be shown by taking the derivative of the LHS of (4.13) with

[11] See the appendix for details.

[12]

$$(1-\nu)\,\frac{1 + (n-1)\,F(z)^n - nF(z)^{n-1}}{(1-F(z))^2}\,\widetilde{p}\left(f(z)^2 + (1 - F(z))\,f'(z)\right) > 0$$

where $z = \bar{x} - p^c - \widetilde{p}$.

[13] $p^m = \arg\max_p \{p \Pr[\varepsilon > p]\}$

[14]

$$\begin{aligned} &1 - F(p^m)^n - \nu\frac{1-F(\bar{x})^n}{1-F(\bar{x})} f(\bar{x})\,p^m + \nu n p^m \int_0^{\bar{x}-p^m} F(\varepsilon + p^m)^{n-1} f'(\varepsilon + p^m)\,d\varepsilon \\ &-(1-\nu)\,\frac{1-F(p^m)^n}{1-F(p^m)} f(p^m)\,p^m < 1 - F(p^m)^n - \frac{1-F(p^m)^n}{1-F(p^m)} f(p^m)\,p^m = 0 \end{aligned}$$

where the inequality has been obtained by setting $\bar{x} = p^m$ because the derivative decreases with $\bar{x}$.

respect to ν. This derivative is negative.[15] The derivative of $\widetilde{\pi}$ with respect to $\widetilde{p}$ is positive because $\widetilde{p} < p^c$. Consequently, the first period punishment profit increases with the search cost.

A cartel is sustainable if the discounted deviation loss is higher than the deviation gain. Firms set different prices in the first punishment period and the subsequent periods. Therefore, the cartel stability condition in (4.12) needs to be modified. The first period deviation loss equals $\pi^c - \widetilde{\pi}$ and later the difference between the collusive profit and the competitive profit is the same as in section 4.4, i.e. $\pi^c - \pi^*$. Then the cartel stability condition is as follows:

$$\pi^d - \pi^c \leq \delta\left(\pi^c - \widetilde{\pi}\right) + \frac{\delta^2}{1-\delta}\left(\pi^c - \pi^*\right) \tag{4.15}$$

The first period punishment profit is different from the competitive profit π^*. Hence, the total deviation loss on the RHS of (4.15) is different from the total deviation loss on the RHS of (4.12). Thus, the new critical discount factor value that makes (4.15) an equation is not the same as $\widehat{\delta}$. We label the new critical discount factor value $\widetilde{\delta}$.

$$\widetilde{\delta}^2\left(\widetilde{\pi} - \pi^*\right) + \widetilde{\delta}\left(\pi^d - \widetilde{\pi}\right) + \left(\pi^c - \pi^d\right) = 0 \tag{4.16}$$

If the fraction of consumers does not get informed about the deviation at the end of the deviation period then the critical discount factor is greater than $\widehat{\delta}$. The punishment price in the first period is higher than p^*. Hence, $\widetilde{\pi} > \pi^*$, which implies that the total punishment is softer. As a result, a higher value of the discount factor than $\widehat{\delta}$ is needed to make (4.15) an equality.

For a formal proof that $\widetilde{\delta} > \widehat{\delta}$ we take the derivatives of the LHS of (4.16) with respect to $\widetilde{\pi}$ and with respect to $\widetilde{\delta}$. The derivative with respect to $\widetilde{\delta}$ is positive and the derivative with respect to $\widetilde{\pi}$ is negative.[16]. Thus, according to the Implicit function theorem, $\widetilde{\delta}$ increases with $\widetilde{\pi}$. If we set $\widetilde{\pi} = \pi^*$ in (4.16) then we get the

[15] The derivative of the LHS of (4.13) with respect to ν is as follows

$$-\widetilde{p}\frac{1-F\left(\bar{x}\right)^n}{1-F\left(\bar{x}\right)}f\left(\bar{x}\right) + \widetilde{p}\frac{1-F\left(\bar{x}-p^c+\widetilde{p}\right)^n}{1-F\left(\bar{x}-p^c+\widetilde{p}\right)}f\left(\bar{x}-p^c+\widetilde{p}\right) + n\widetilde{p}\int_{\bar{x}-p^c}^{\bar{x}-\widetilde{p}} F\left(\varepsilon+\widetilde{p}\right)^{n-1} f'\left(\varepsilon+\widetilde{p}\right)d\varepsilon \tag{4.14}$$

This derivative decreases with p^c because its derivative with respect to p^c is negative:

$$-\widetilde{p}\frac{1+(n-1)F\left(z\right)^n - nF\left(z\right)^{n-1}}{\left(1-F\left(z\right)\right)^2}\left(f^2\left(z\right) + \left(1-F\left(z\right)\right)f'\left(z\right)\right) < 0$$

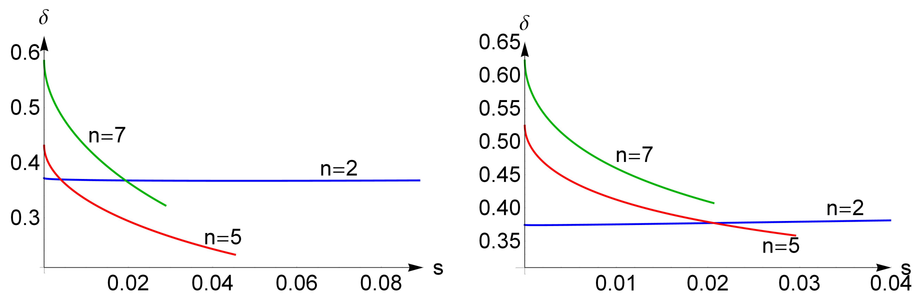
were $z = \bar{x} - p^c + \widetilde{p}$. Then (4.14) is less than if we set $p^c = \widetilde{p}$ which makes (4.14) equal to zero. As a result the LHS of (4.13) decreases with ν.

[16] The derivative with respect to $\widetilde{\delta}$ is $2\widetilde{\delta}\left(\widetilde{\pi} - \pi^*\right) + \pi^d - \widetilde{\pi} > 0$, and The derivative with respect to $\widetilde{\pi}$ is $\widetilde{\delta}^2 - \widetilde{\delta} < 0$

condition (4.12) which defines $\widehat{\delta}$. Consequently, $\widetilde{\delta} > \widehat{\delta}$.

The LHS of (4.16) is a second degree convex polynomial in $\widetilde{\delta}$ with two roots. It is difficult to show how the critical discount factor varies with the search cost analytically. Thus, we use numerical simulation for further analysis. The simulation results suggest that both roots of (4.16) are positive. However, one root is higher than one. Hence, we use the smaller root for the analysis of cartel stability.

The profits of firms increase with the search cost in all punishment periods. Therefore, the deviation loss decreases with the search cost even if not all consumers observe the deviation price p^d immediately. The deviation gain decreases with the search cost too. If ε is distributed uniformly then the deviation gain is more sensitive to the changes of s than the deviation loss. Therefore, the critical discount factor $\widetilde{\delta}$ decreases with the search cost (see Figure 4.2a). However, if ε is distributed according to the exponential-type distribution and $n = 2$ then the critical discount factor decreases with s for small s values and increases with s if s is sufficiently high. (see Figure 4.2b).



(a) $\varepsilon \sim U(0,1)$ (b) $F(\varepsilon) = \frac{e^{\varepsilon}-1}{e-1}$

Figure 4.2. The values of $\widetilde{\delta}$ for different values of s

4.6 Product differentiation

It has been noticed by Perloff and Salop (1985) that "*an increase in preference intensity ... raises the equilibrium price.*"[17] High market power leads to high firms' profits. Thus, collusion becomes less attractive if the substitutability between products decreases. In addition, it has been shown in the collusion literature that high product differentiation decreases the deviation gain. However, the low substitutability between products "*limits the severity of price wars and, thus the firms' ability to punish*

[17] See Proposition 1 in Perloff and Salop (1985)

a potential deviation".[18] Consequently, the effect of product differentiation on the cartel stability is ambiguous and is closely related to the model assumptions.

We analyze the effect of product differentiation on the cartel stability with costly consumer search in this section. For this reason, we introduce a new variable $\mu > 0$ and assume that the utility of consumer i, who buys product j equals

$$u_{ij} = \mu\varepsilon_{ij} - p_j$$

If products are highly differentiated then μ is high and the utilities from two different products differ a lot. On the contrary, if the value of μ is small then the difference $\mu\left|\varepsilon_{ji} - \varepsilon_{li}\right|$ is small for all $j \neq l$. With the purpose of algebraic simplicity of the proofs the assumption that both the firms and the consumers get informed about the prices in the end of every period is maintained in the analysis.

The introduction of a new variable slightly affects the expressions of the search rule and the pay-off functions that have been derived in sections 4.2, 4.3 and 4.4. We start with the changes of the search rule that has been defined in (4.3), and later proceed with the analysis of profits and prices.

The LHS of equation (4.3) changes slightly if $\mu \neq 1$. More particularly, is becomes

$$\mu \int_{\varepsilon_j - p_j/\mu + p_l/\mu}^{\bar{\varepsilon}} \left(\varepsilon_l - \widetilde{x}\right) dF\left(\varepsilon_l\right) = s$$

where $\widetilde{x} = \varepsilon_j - \frac{p_j - p_l}{\mu}$. If we denote the price of n-variety monopolist by p^c then $\widetilde{x} \geq p^c/\mu$.

Let us consider a firm j, which looks for a profit maximizing price $p_j \neq p^*$ in a competitive market. Then $\Delta_\mu = \left(p^* - p_j\right)/\mu$. A consumer searches beyond firm j if $\varepsilon_j < \widetilde{x} - \Delta_\mu$. If the inequality is reversed then the consumer terminates her search in firm j. If the consumer searches all firms, then she returns to shop j and buys here if the utility here is the highest in the market and is non-negative. As a result, a returning consumer buys from firm j if $\varepsilon > p_j/\mu$ and $\varepsilon_j = \max\left\{\varepsilon_l, 0\right\}_{l=1,\dots n}$. To sum up, if $\mu \neq 1$ then the threshold values $\bar{x} - \Delta_1$ must be replaced with $\widetilde{x} - \Delta_\mu$ and all prices must be divided by μ in the demand expression in section 4.3.1.

In equilibrium firm j does not want to deviate from consumers' expectations and charges price p^*. Thus, the first order condition of a firm looks similar to equation (4.6):

[18] Ivaldi et al. (2003a).

$$1 - \frac{1 - F(\widetilde{x})^n}{1 - F(\widetilde{x})} \frac{p^*}{\mu} f(\widetilde{x}) - F\left(\frac{p^*}{\mu}\right)^n + n\frac{p^*}{\mu} \int_{p^*/\mu}^{\widetilde{x}} F(\varepsilon)^{n-1} f'(\varepsilon)\, d\varepsilon = 0 \qquad (4.17)$$

The new variable μ enters the first order condition (4.17). Therefore, it has an impact on p^*. The equation (4.17) looks very similar to equation (4.6). The only difference is that $\bar{x}$ is replaced with $\widetilde{x}$ and p^* is replaced with p^*/μ. As a result, the LHS of (4.17) decreases with p^*/μ and decreases with $\widetilde{x}$. Therefore, for any fixed value of μ the equilibrium price p^* increases with the search cost.

The calculation of the effect of μ on the competitive price is not that trivial. The degree of product differentiation has a twofold effect on p^*. Firstly, if μ increases then consumers are more loyal to the firms and the sellers have incentives to raise their prices. Secondly, a high degree of product differentiation encourages consumers to search more because $\widetilde{x}$ increases with μ. More searching customers imply higher competitive pressure on firms and push the equilibrium price down. Furthermore, there is the third effect of μ in our model. An increase in μ leads to an increase in the mean of $\mu\varepsilon$ in our model because we require that $\varepsilon \geq 0$. The higher mean implies that consumers value all the varieties more in general. As a result, the firms have incentives to increase their prices. The total effect depends on the sum of three effects and it is sensitive to the assumptions about the distribution function and the range of values of ε.[19]

Claim 4.4. *Assume that either (1) $s \to 0$ or (2) $\varepsilon \sim U[0, 1]$. Then the competitive price p^* increases with the degree of product differentiation.*

The competitive profit π^* can be written as

$$\pi^* = \frac{p^*}{n}\left(1 - F\left(\frac{p^*}{\mu}\right)^n\right)$$

This profit increases with the degree of product differentiation if the equilibrium price increases with μ because it increases with p^* and the direct effect of μ is also positive.

[19] See Anderson and Renault (1999) for more details.
Perloff and Salop (1985) suggest that product differentiation can be modeled differently. They propose to assume that the match value is distributed according to a symmetric mean preserving distribution with a positive variance. An increase in the variance makes the tails of the distribution thicker, which implies that products a more differentiated. If the variance decreases then the majority of ε values are concentrated around the mean, which implies that products are more homogeneous. If product differentiation is modeled in this way then an equilibrium price may decrease if products become more heterogeneous.

The cartel serves the consumers whose valuation for at least one variety is more than or equal to μp^c. Hence the joint profit function is

$$\Pi^c = p^c \left(1 - F\left(\frac{p^c}{\mu}\right)^n\right) \tag{4.18}$$

and the price p^c is defined by the following equation

$$1 - F\left(\frac{p^c}{\mu}\right)^n - n\frac{p^c}{\mu}F\left(\frac{p^c}{\mu}\right)^{n-1} f\left(\frac{p^c}{\mu}\right) = 0 \tag{4.19}$$

The collusive price increases with the degree of product differentiation because the LHS of the first order condition of a monopolist increases with μ. This increase is mainly driven by the increase in the mean of $\mu\varepsilon$. The positive effect of μ on the collusive price has a positive effect on the collusive profit too. Every cartel member gets higher profit for any fixed search cost and the number of firms if the degree of product differentiation increases.

The increased product differentiation does not remove the incentives to deviate from the collusive set-up unilaterally. A deviating firm sets $p^d < p^c$ if it maximizes its own profit, given that other sellers set p^c in their shops. If $\mu \neq 1$ then the payoff function and the first order condition of the deviant are very similar to the ones that have been derived in section 4.4. The only difference is that both prices p^c and p^d must be divided by μ and $\bar{x}$ must be replaced with $\widetilde{x}$.

Claim 4.5. *If (1) $s \to 0$ or (2) $\varepsilon \sim U(0, 1)$ then the deviation price p^d increases with the degree of product differentiation*

A deviant prefers to set a high price to its customers because they have high valuations for the deviant's product if μ is high. However, the difference in prices becomes less important when products get more differentiated. Therefore, the firm must reduce its price a lot if it wants to steal some demand from its rivals. The positive effect of μ is stronger than the negative. Therefore, the deviation price increases with μ.

The derivative of π^d with respect to μ can be written as follows

$$\frac{\partial \pi^d}{\partial \mu} = \frac{\partial \pi^d}{\partial \mu} + \frac{\partial \pi^d}{\partial \widetilde{x}}\frac{\partial \widetilde{x}}{\partial \mu} + \frac{\partial \pi^d}{\partial p^c}\frac{\partial \pi^d}{\partial \mu} \tag{4.20}$$

The effect of μ that comes via p^d cancels out because $\partial \pi^d / \partial p^d = 0$. If products become more differentiated then a consumer searches more. This has a negative effect on the demand of the deviant because consumers are more picky. However, more intensive search implies that more consumers observe the deviation price. In addition, the collusive price increases with μ. Then the total effect of the degree of product differentiation depends on whether the positive or negative effect is stronger.

Claim 4.6. *If (1) $s \to 0$ or (2) $\varepsilon \sim U(0, 1)$ then the deviation profit π^d increases with the degree of product differentiation.*

The incentives to deviate vary with the degree of product heterogeneity. All profits increase with μ if ε is distributed uniformly or the search cost is very close to zero. Therefore, it is not explicitly clear whether the critical discount factor increases or decreases with μ. The expressions of the derivative of the deviation gain and the deviation loss with respect to μ are complicated. Therefore, we proceed with the case $\varepsilon \sim U(0, 1)$ in the analysis of $\partial\widehat{\delta}/\partial\mu$.

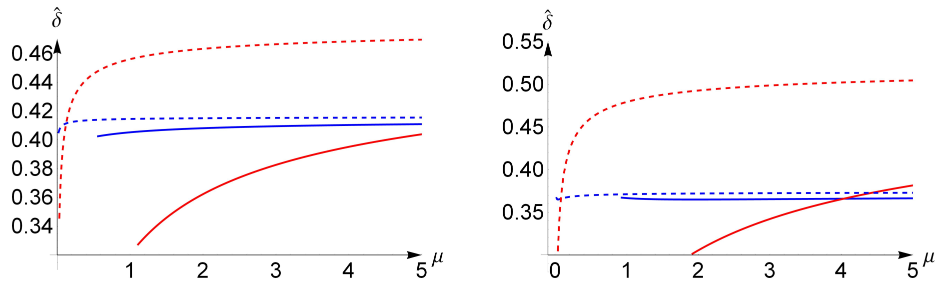
Proposition 4.2. *Assume that $\varepsilon \sim U(0,1)$. Then*
(A) if $s \to 0$ then the critical discount factor does not depend on μ;
(B) if $s \to \bar{s}_n$ then the critical discount factor increases with μ.

We simulated the values of $\widehat{\delta}$ for several values of μ, n and s, given that $\varepsilon \sim U(0,1)$ and $F(\varepsilon) = \frac{e^{\varepsilon}-1}{e-1}$. The simulation results show that the critical discount factor increases with μ for any size or search costs if the match value is distributed uniformly (Figure 4.3a). This happens because the deviation profit increases with the product heterogeneity parameter faster than the competitive profit. Therefore, it is harder to sustain a cartel if product get more differentiated. The same result holds for the exponential distribution of ε if $n > 2$. (Figure 4.3b). If there are two firms in the market and the match value is distributed exponentially, then for small values of μ the deviation loss increases faster with μ than the deviation gain. Then a cartel becomes more stable if the product differentiation increases and consumer search is costly. However, if μ is sufficiently high then collusion is more stable if μ increases for any number of firms.

Proposition 4.3. *The preference intensity parameter does not have an effect on the sign of $\partial\widehat{\delta}/\partial s$ if $s \leq \bar{s}_n$.*

The preference intensity parameter μ does not affect the result of proposition 4.1 because the parameter of product heterogeneity works as a scaling factor in the expressions of the first order conditions and profits. As a result, the derivatives of prices and profits with respect to $\widetilde{x}$ are similar to the derivatives that have been derived for the case $\mu = 1$.[20] Hence, the critical discount factor $\widehat{\delta}$ increases (or

[20] See the formal proof of the proposition in the appendix.



(a) $\varepsilon \sim U(0,1)$ (b) $F(\varepsilon) = \frac{e^{\varepsilon}-1}{e-1}$

Figure 4.3. The values of $\widehat{\delta}$ for different values of n, μ, and s (blue lines $n = 2$, red lines $n = 5$, solid lines $s = 0.05$, dashed lines $s = 0.001$)

decreases) with the search cost no matter the value of μ. However, the value of the the derivative $\partial\widehat{\delta}/\partial\widetilde{x}$ depends on the product heterogeneity parameter as well as the value of $\widehat{\delta}$.

4.7 Conclusions

Both the EC and the FTC acknowledge that collusion is easier to sustain if firms can monitor each other easily.[21]. The findings of the economics literature show that market transparency from the point of view of firms decreases the deviation gain and makes the punishment harder, because the probability to gain from deviating more than one period decreases if sellers can observe the deviation better.

However, market transparency on the consumer side works in the opposite way. This paper has studied the effect of the search costs on collusion stability. An increase in consumer search costs makes a market less transparent because consumers search less and observe less offers. We have shown that a cartel often is more stable if the search costs increase. This happens because the deviation is less attractive if less consumers observe it. The punishment becomes less severe if the search cost increases because the competitive profit increases with the search cost. However, the effect of costly search is stronger on the deviation gain than the deviation loss. Thus, collusion is more stable if the search costs increase.

Our findings are in line with the findings of Schultz (2005). He analyzed the

[21] *"The market typically is more vulnerable to coordinated conduct if each competitively important firm's significant competitive initiatives can be promptly and confidently observed by that firm's rivals"*, Federal Trade Commission (2010)
"...the coordinating firms must be able to monitor to a sufficient degree whether the terms of coordination are being adhered to..." European Commission (2010a)

effect of market transparency on cartel stability by introducing the fraction of uniformed consumers in a Hotelling model. He shows that that the minimum discount factor value, above which a cartel is stable, decreases with the fraction of uninformed consumers. In other words, a cartel gets less stable if market becomes more transparent. However, our model differs from the model of Schultz because we introduce the search costs in our model instead of fixing the consideration set of a consumer exogenously.

Collusion is easier to sustain if a market is more transparent on the side of firms and is less transparent from the point of view of consumers. Unfortunately Stigler (1964) has observed that *"no one has yet invented a way to advertise price reductions which brings them to the attention of numerous customers but not to that of any rival."* Therefore, it is a difficult task for competition authorities to identify the right amount of market transparency.

4.A Appendix

The second derivative of π_j with respect to p_j. The first order derivative of π_j with respect to p_j may be written as follows

$$
\begin{aligned}
n\frac{\partial \pi_j}{\partial p_j} &= \frac{1-F(\bar{x})^n}{1-F(\bar{x})}\left(1-F\left(\bar{x}-p^*+p_j\right)-p_j f\left(\bar{x}-p^*+p_j\right)\right)\\
&+n\int_{p_j}^{\bar{x}-p^*+p_j} F\left(\varepsilon-p_j+p^*\right)^{n-1} f(\varepsilon)\,d\varepsilon\\
&+np_j\int_{p_j}^{\bar{x}-p^*+p_j} F\left(\varepsilon-p_j+p^*\right)^{n-1} f'(\varepsilon)\,d\varepsilon
\end{aligned}
$$

Then

$$
\begin{aligned}
n\frac{\partial \pi_j^2}{\partial p_j^2} &= -\frac{1-F(\bar{x})^n}{1-F(\bar{x})}\left(2f\left(\bar{x}-p^*+p_j\right)+p_j f'\left(\bar{x}-p^*+p_j\right)\right)\\
&+nF(\bar{x})^{n-1} f\left(\bar{x}-p^*+p_j\right)-nF(p^*)^{n-1} f\left(p_j\right)\\
&-n(n-1)\int_0^{\bar{x}-p^*} F(\varepsilon+p^*)^{n-2} f(\varepsilon+p^*) f\left(\varepsilon+p_j\right)d\varepsilon\\
&+n\int_0^{\bar{x}-p^*} F(\varepsilon+p^*)^{n-1} f'\left(\varepsilon+p_j\right)d\varepsilon+np_jF(\bar{x})^{n-1} f'\left(\bar{x}-p^*+p_j\right)\\
&-n(n-1)p_j\int_0^{\bar{x}-p^*} F(\varepsilon+p^*)^{n-2} f(\varepsilon+p^*) f'\left(\varepsilon+p_j\right)d\varepsilon\\
&-np_jF(p^*)^{n-1} f'\left(p_j\right)
\end{aligned}
$$

If we apply integration by parts then we get the following equality

$$
\begin{aligned}
&n\int_0^{\bar{x}-p^*} F(\varepsilon+p^*)^{n-1} f'\left(\varepsilon+p_j\right)d\varepsilon = nF(\bar{x})^{n-1} f\left(\bar{x}-p^*+p_j\right)-nF(p^*)^{n-1} f\left(p_j\right)\\
&-n(n-1)\int_0^{\bar{x}-p^*} F(\varepsilon+p^*)^{n-2} f(\varepsilon+p^*) f\left(\varepsilon+p_j\right)d\varepsilon
\end{aligned}
$$

We plug the last result into the expression $n\frac{\partial \pi_j^2}{\partial p_j^2}$ and get

$$
\begin{aligned}
n\frac{\partial \pi_j^2}{\partial p_j^2} &= -\frac{1+(n-1)F(\bar{x})^n-nF(\bar{x})^{n-1}}{1-F(\bar{x})}\left(2f\left(\bar{x}-p^*+p_j\right)+p_j f'\left(\bar{x}-p^*+p_j\right)\right)\\
&-nF(p^*)^{n-1}\left(2f\left(p_j\right)+p_j f'\left(p_j\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&- 2n\left(n-1\right)\int_{0}^{\bar{x}-p^{*}} F\left(\varepsilon+p^{*}\right)^{n-2} f\left(\varepsilon+p^{*}\right) f\left(\varepsilon+p_{j}\right) d\varepsilon \\
&- n\left(n-1\right) p_{j}\int_{0}^{\bar{x}-p^{*}} F\left(\varepsilon+p^{*}\right)^{n-2} f\left(\varepsilon+p^{*}\right) f^{'}\left(\varepsilon+p_{j}\right) d\varepsilon < 0
\end{aligned}
$$

The fraction $\frac{1+(n-1)F(\bar{x})^{n}-nF(\bar{x})^{n-1}}{1-F(\bar{x})}$ is positive because $1+\left(n-1\right)F\left(\bar{x}\right)^{n}-nF\left(\bar{x}\right)^{n-1}\geq 1+\left(n-1\right)1^{n}-n1^{n-1}=0$ and $2f\left(p\right)+pf^{'}\left(p\right)>0$. ■

The derivative of the LHS of (4.6) with respect to p^{*}.

$$
\begin{aligned}
&-\frac{1-F\left(\bar{x}\right)^{n}}{1-F\left(\bar{x}\right)}f\left(\bar{x}\right)-nF\left(p^{*}\right)^{n-1}f\left(p^{*}\right)+n\int_{p^{*}}^{\bar{x}}F\left(\varepsilon\right)^{n-1}f^{'}\left(\varepsilon\right)d\varepsilon \\
&-nF\left(p^{*}\right)^{n-1}f^{'}\left(p^{*}\right)p^{*}=-\frac{1+\left(n-1\right)F\left(\bar{x}\right)^{n}-nF\left(\bar{x}\right)^{n-1}}{1-F\left(\bar{x}\right)}f\left(\bar{x}\right) \\
&-nF\left(p^{*}\right)^{n-1}\left(2f\left(p^{*}\right)+f^{'}\left(p^{*}\right)p^{*}\right) \\
&-n\left(n-1\right)\int_{p^{*}}^{\bar{x}}F\left(\varepsilon\right)^{n-2}f\left(\varepsilon\right)^{2}d\varepsilon<0
\end{aligned}
$$

where the second equality has been obtained because $\int_{p^{*}}^{\bar{x}}F\left(\varepsilon\right)^{n-1}f^{'}\left(\varepsilon\right)d\varepsilon = F\left(\bar{x}\right)^{n-1}f\left(\bar{x}\right)-F\left(p^{*}\right)^{n-1}f\left(p^{*}\right)-\int_{p^{*}}^{\bar{x}}f\left(\varepsilon\right)dF\left(\varepsilon\right)^{n-1};$ and $1+\left(n-1\right)F\left(\bar{x}\right)^{n}-nF\left(\bar{x}\right)^{n-1}>1+\left(n+1\right)1^{n}-n1^{n-1}=0.$ ■

The derivative of the LHS of (4.6) with respect to $\bar{x}$.

$$
\begin{aligned}
&-\frac{1+\left(n-1\right)F\left(\bar{x}\right)^{n}-nF\left(\bar{x}\right)^{n-1}}{\left(1-F\left(\bar{x}\right)\right)^{2}}p^{*}f\left(\bar{x}\right)^{2}-\frac{1-F\left(\bar{x}\right)^{n}}{1-F\left(\bar{x}\right)}p^{*}f^{'}\left(\bar{x}\right)+nF\left(\bar{x}\right)^{n-1}p^{*}f^{'}\left(\bar{x}\right) \\
&=-\frac{1+\left(n-1\right)F\left(\bar{x}\right)^{n}-nF\left(\bar{x}\right)^{n-1}}{\left(1-F\left(\bar{x}\right)\right)^{2}}p^{*}\left(f\left(\bar{x}\right)^{2}+\left(1-F\left(\bar{x}\right)\right)f^{'}\left(\bar{x}\right)\right)<0
\end{aligned}
$$

The inequality has been obtained because $\left[\frac{f}{1-F}\right]^{'}>0.$ ■

The proof that p^{c} increases with n. We rewrite the first order condition (4.10) as $K\left(p^{c},n\right)$:

$$
K\left(p^{c},n\right)\equiv\int_{p^{c}}^{1}\left(\frac{F\left(\varepsilon\right)}{F\left(p^{c}\right)}\right)^{n-1}f\left(\varepsilon\right)d\varepsilon-f\left(p^{c}\right)p^{c}=0
$$

The derivative of $K\left(p^{c},n\right)$ with respect to n is non-negative because $\varepsilon\geq p^{c}$.

Meanwhile the derivative of $K(p^c, n)$ with respect to p^c is negative:

$$\begin{aligned}\frac{\partial K(p^c,n)}{\partial p^c} &= -\int_{p^c}^{1}\left[(n-1)\frac{F(\varepsilon)^{n-1}}{F(p^c)^n} f(\varepsilon) f(p^c)\right] d\varepsilon - 2f(p^c) - p^c f'(p^c) \\ &= \frac{1-n}{n}\frac{\left[1-F(p^c)^n\right] f(p^c)}{F(p^c)^n} - 2f(p^c) - p^c f'(p^c) < 0\end{aligned}$$

Thus, by the Implicit function theorem we get that $\partial p^c/\partial n > 0$. ■

Proof of claim 4.1. Firstly we need to prove that the LHS of 4.11 decreases with p^d. Afterwards we show that $p^d < p^c$. And finally we show that $p^* \leq p^d$.

Let us denote the LHS of (4.11) by $V\left(p^d\right)$. Then $\partial V\left(p^d\right)/\partial p^d < 0$. The derivative is similar to the second derivative of π_j with respect to p_j. Therefore, omit it here.

Let us check the sign of $V(p^c)$.

$$V(p^c) = 1 - \frac{1-F(\bar{x})^n}{1-F(\bar{x})} p^c f(\bar{x}) - F(p^c)^n + n\int_0^{\bar{x}-p^c} F(\varepsilon+p^c)^{n-1} p^c f'(\varepsilon+p^c)\, d\varepsilon$$

The form of $V(p^c)$ is similar to the first order condition (4.6). It has been shown that the LHS of (4.6) decreases with p^*. Moreover, $p^c > p^m > p^*$. Thus, $V(p^c) < 0$ and $p^d < p^c$.

If $f(y) + pf'(y) > 0$, $\forall p < y$ is satisfied then the derivative of the LHS of (4.11) with respect to p^c is positive:

$$\begin{aligned}&\frac{1+(n-1)F(\bar{x})^n - nF(\bar{x})^{n-1}}{1-F(\bar{x})}\left(f\left(\bar{x}-p^c+p^d\right) + p^d f'\left(\bar{x}-p^c+p^d\right)\right) \\ &+ n(n-1)\int_0^{\bar{x}-p^c} F(\varepsilon+p^c)^{n-2} f(\varepsilon+p^c)\left(f\left(\varepsilon+p^d\right) + p^d f'\left(\varepsilon+p^d\right)\right) d\varepsilon > 0\end{aligned}$$

Thus, if $p^c > p^*$ then $p^d > p^*$. ■

Proof of claim 4.2. We apply the Implicit function theorem to prove this lemma. It has been shown that the LHS of (4.11) decreases with p^d. Thus, we need to show that the LHS of (4.11) decreases with $\bar{x}$. The derivative of the LHS of (4.11) with respect to $\bar{x}$ is as follows:

$$\frac{1+(n-1)F(\bar{x})^n - nF(\bar{x})^{n-1}}{(1-F(\bar{x}))^2} f(\bar{x})\left(1 - F\left(\bar{x}-p^c+p^d\right) - p^d f\left(\bar{x}-p^c+p^d\right)\right)$$

$$-\frac{1+(n-1)F(\bar{x})^{n}-nF(\bar{x})^{n-1}}{1-F(\bar{x})}\left(f\left(\bar{x}-p^{c}+p^{d}\right)+p^{d}f^{'}\left(\bar{x}-p^{c}+p^{d}\right)\right) \tag{4.A.1}$$

If $f(y)+pf^{'}(y)>0,\ \forall p<y$ then from the first order condition (4.11) we get that $1-F\left(\bar{x}-p^{c}+p^{d}\right)-p^{d}f\left(\bar{x}-p^{c}+p^{d}\right)$ is negative, which completes the proof. ■

Proof of claim 4.3. The deviating firm sets p^{d} such that the derivative of π^{d} equals zero. Therefore, only the direct effect of the search cost matters. Then

$$\begin{aligned}\frac{n}{p^{d}}\frac{\partial\pi^{d}}{\partial\bar{x}}&=\frac{1+(n-1)F(\bar{x})^{n}-nF(\bar{x})^{n-1}}{(1-F(\bar{x}))^{2}}f(\bar{x})\left(1-F\left(\bar{x}-p^{c}+p^{d}\right)\right)\\&-\frac{1-F(\bar{x})^{n}}{1-F(\bar{x})}f\left(\bar{x}-p^{c}+p^{d}\right)+nF(\bar{x})^{n-1}f\left(\bar{x}-p^{c}+p^{d}\right)\\&=\frac{1+(n-1)F(\bar{x})^{n}-nF(\bar{x})^{n-1}}{(1-F(\bar{x}))}\left(1-F\left(\bar{x}-p^{c}+p^{d}\right)\right)\left(\frac{f(\bar{x})}{1-F(\bar{x})}\right.\\&\left.-\frac{f\left(\bar{x}-p^{c}+p^{d}\right)}{1-F\left(\bar{x}-p^{c}+p^{d}\right)}\right)>0\end{aligned}$$

Thus, the profit of a deviating firm increases with $\bar{x}$ or decreases with the search cost. ■

Proof of proposition 4.1. If (4.12) holds with the equality then the critical discount factor value $\widehat{\delta}$ can be written in the following way

$$\widehat{\delta}=\frac{\pi^{d}-\pi^{c}}{\pi^{d}-\pi^{*}} \tag{4.A.2}$$

The profit of a coalition member does not depend on the search cost. Thus, the derivative of the critical discount factor with respect to $\bar{x}$ depends only on the changes of π^{d} and π^{*}.

$$\begin{aligned}\frac{\partial\widehat{\delta}}{\partial\bar{x}}&=\frac{1}{\left(\pi^{d}-\pi^{*}\right)^{2}}\left[\frac{\partial\pi^{d}}{\partial\bar{x}}\left(\pi^{d}-\pi^{*}\right)-\left(\frac{\partial\pi^{d}}{\partial\bar{x}}-\frac{\partial\pi^{*}}{\partial\bar{x}}\right)\left(\pi^{d}-\pi^{c}\right)\right]\\&=\frac{1}{\left(\pi^{d}-\pi^{*}\right)^{2}}\left[\frac{\partial\pi^{d}}{\partial\bar{x}}\left(\pi^{c}-\pi^{*}\right)+\frac{\partial\pi^{*}}{\partial\bar{x}}\left(\pi^{d}-\pi^{c}\right)\right]\end{aligned} \tag{4.A.3}$$

The denominator of the first fraction in (4.A.3) is positive. Hence,

$$\operatorname{sgn}\left[\frac{\partial\widehat{\delta}}{\partial\bar{x}}\right]=\operatorname{sgn}\left[\frac{\partial\pi^{d}}{\partial\bar{x}}\left(\pi^{c}-\pi^{*}\right)+\frac{\partial\pi^{*}}{\partial\bar{x}}\left(\pi^{d}-\pi^{c}\right)\right] \tag{4.A.4}$$

If ε is distributed uniformly then from the proof of claim 4.3 we have that

$$\begin{aligned}\frac{\partial\pi^{d}}{\partial\bar{x}} &= \frac{p^{d}}{n}\frac{1+(n-1)\,\bar{x}^{n}-n\bar{x}^{n-1}}{(1-\bar{x})^{2}}\left(1-\bar{x}+p^{c}-p^{d}\right)\\ &- \frac{p^{d}}{n}\frac{1+(n-1)\,\bar{x}^{n}-n\bar{x}^{n-1}}{1-\bar{x}}=\frac{p^{d}}{n}\frac{1+(n-1)\,\bar{x}^{n}-n\bar{x}^{n-1}}{(1-\bar{x})^{2}}\left(p^{c}-p^{d}\right)\end{aligned}$$

The competitive equilibrium profit may be written as follows

$$\pi^{*}=\left(p^{*}\right)^{2}\left.\frac{\partial q_{j}}{\partial p_{j}}\right|_{p_{j}=p^{*}}=\frac{\left(p^{*}\right)^{2}}{n}\frac{1-\bar{x}^{n}}{1-\bar{x}}$$

Then the derivative of the competitive profit with respect to $\bar{x}$ may be written as

$$\frac{\partial\pi^{*}}{\partial\bar{x}}=\frac{\left(p^{*}\right)^{2}}{n}\frac{1+(n-1)\,\bar{x}^{n}-n\bar{x}^{n-1}}{(1-\bar{x})^{2}}\left(1-\frac{2\frac{1-\bar{x}^{n}}{1-\bar{x}}}{\frac{1-\bar{x}^{n}}{1-\bar{x}}+n\left(p^{*}\right)^{n-1}}\right)$$

Note that both the derivative of π^{d} with respect to $\bar{x}$ and the derivative π^{*} with respect to $\bar{x}$ have the same positive element $\frac{1}{n}\frac{1+(n-1)\bar{x}^{n}-n\bar{x}^{n-1}}{(1-\bar{x})^{2}}$. The sign of the RHS of (4.A.4) does not change if we divide it by this expression. Therefore, we explore the sign of the expression $\phi\left(\bar{x},p^{*},n\right)$ further on.

$$\begin{aligned}\phi\left(\bar{x},p^{*},n\right) &\equiv p^{d}\left(p^{c}-p^{d}\right)\left(p^{c}\left(1-\left(p^{c}\right)^{n}\right)-\left(p^{*}\right)^{2}\frac{1-\bar{x}^{n}}{1-\bar{x}}\right)\\ &+ \left(p^{*}\right)^{2}\left(1-\frac{2\frac{1-\bar{x}^{n}}{1-\bar{x}}}{\frac{1-\bar{x}^{n}}{1-\bar{x}}+n\left(p^{*}\right)^{n-1}}\right)\left(\left(p^{d}\right)^{2}\frac{1-\bar{x}^{n}}{1-\bar{x}}-p^{c}\left(1-\left(p^{c}\right)^{n}\right)\right)\end{aligned}$$

The uniform distribution of ε implies that

$$p^{d}=\frac{1}{2}\left(p^{c}+\frac{1-\bar{x}}{1-\bar{x}^{n}}\left(1-\left(p^{c}\right)^{n}\right)\right)$$

We also use the fact that $1-\left(p^{c}\right)^{n}=n\left(p^{c}\right)^{n}$

Hence,

$$n\left(\pi^{d}-\pi^{c}\right)=\frac{1}{4}\left(\left(p^{c}\right)^{2}+2\frac{1-\bar{x}}{1-\bar{x}^{n}}n\left(p^{c}\right)^{n+1}\right.$$
$$\left.+\left(\frac{1-\bar{x}}{1-\bar{x}^{n}}\right)^{2}n^{2}\left(p^{c}\right)^{2n}\right)\frac{1-\bar{x}^{n}}{1-\bar{x}}-n\left(p^{c}\right)^{n+1}$$
$$=\frac{1}{4}\frac{1-\bar{x}^{n}}{1-\bar{x}}\left(\left(p^{c}\right)^{2}-2\frac{1-\bar{x}}{1-\bar{x}^{n}}n\left(p^{c}\right)^{n+1}+\left(\frac{1-\bar{x}}{1-\bar{x}^{n}}\right)^{2}n^{2}\left(p^{c}\right)^{2n}\right)$$
$$=\frac{1}{4}\frac{1-\bar{x}^{n}}{1-\bar{x}}\left(p^{c}-\frac{1-\bar{x}}{1-\bar{x}^{n}}n\left(p^{c}\right)^{n}\right)^{2}=\frac{1-\bar{x}^{n}}{1-\bar{x}}\left(p^{c}-p^{d}\right)^{2}$$

The collusive price is higher than the deviation price. Therefore, if we divide $\phi\left(\bar{x},p^{*},n\right)$ by $p^{c}-p^{d}$ then the new expression will have the same sign as $\phi\left(\bar{x},p^{*},n\right)$. We denote this new expression by $\phi_{1}\left(\bar{x},p^{*},n\right)$

$$\phi_{1}\left(\bar{x},p^{*},n\right)=p^{d}\left(n\left(p^{c}\right)^{n+1}\right.$$
$$\left.-\left(p^{*}\right)^{2}\frac{1-\bar{x}^{n}}{1-\bar{x}}\right)+\left(p^{*}\right)^{2}\left(1-\frac{2\frac{1-\bar{x}^{n}}{1-\bar{x}}}{\frac{1-\bar{x}^{n}}{1-\bar{x}}+n\left(p^{*}\right)^{n-1}}\right)\left(p^{c}-p^{d}\right)\frac{1-\bar{x}^{n}}{1-\bar{x}}$$

From the first order condition (4.6) we know that

$$\left(p^{*}\right)^{n-1}=\frac{1}{p^{*}}-\frac{1-\bar{x}^{n}}{1-\bar{x}}=\frac{1}{p^{*}}-\alpha$$

Then we can rewrite $\phi_{1}\left(\bar{x},p^{*},n\right)$ as

$$\phi_{1}\left(\bar{x},p^{*},n\right)\equiv p^{d}\left(n\left(p^{c}\right)^{n+1}-\left(p^{*}\right)^{2}\alpha\right)+\left(p^{*}\right)^{2}\left(\frac{n-p^{*}\alpha\left(n+1\right)}{n-\alpha\left(n-1\right)p^{*}}\right)\left(p^{c}-p^{d}\right)\alpha$$

Now note that

$$\frac{\partial\left(\phi_{1}\left(\bar{x},p^{*},n\right)\right)}{\partial\alpha}=-p^{d}\left(p^{*}\right)^{2}-\left(p^{*}\right)^{2}\frac{2np^{*}}{\left(n-\alpha\left(n-1\right)p^{*}\right)^{2}}\left(p^{c}-p^{d}\right)\alpha$$
$$+\left(p^{*}\right)^{2}\left(\frac{n-p^{*}\alpha\left(n+1\right)}{n-\alpha\left(n-1\right)p^{*}}\right)\left(p^{c}-p^{d}\right)<0$$

The inequality has been obtained because $\frac{\partial\pi^{*}}{\partial\bar{x}}<0$ implies that $\frac{n-p^{*}\alpha(n+1)}{n-\alpha(n-1)p^{*}}<0$.

Additionally,

$$\frac{\partial\left(\phi_1\left(\bar{x},p^*,n\right)\right)}{\partial p^*} = -2p^d p^* \alpha + 2p^* \left(\frac{n-p^*\alpha\left(n+1\right)}{n-\alpha\left(n-1\right)p^*}\right)\left(p^c-p^d\right)\alpha$$
$$-2\left(p^*\right)^2\left(p^c-p^d\right)\frac{\alpha^2 n}{\left(n-\alpha\left(n-1\right)p^*\right)^2} < 0$$

and

$$\frac{\partial\left(\phi_1\left(\bar{x},p^*,n\right)\right)}{\partial p^d} = n\left(\pi^c-\pi^*\right)-\left(p^*\right)^2\alpha\left(\frac{n-p^*\alpha\left(n+1\right)}{n-\alpha\left(n-1\right)p^*}\right) > 0, \quad \frac{\partial p^d}{\partial \alpha} < 0.$$

Therefore,

$$\phi_1\left(\bar{x},p^*,n\right) > \phi_1\left(1,\frac{1}{2},n\right) = \frac{1}{2}\left(p^c+\frac{1}{n+1}\right)\left(\frac{p^c n}{n+1}-\frac{n}{4}\right)$$
$$+\frac{1}{8}\left(\frac{n-\frac{n}{2}\left(n+1\right)}{n-\frac{n}{2}\left(n-1\right)}\right)\left(p^c-\frac{1}{n+1}\right)n \tag{4.A.5}$$

(4.A.5) is the second degree convex polynomial in p^c. The polynomial is at minimum when

$$p^c = \frac{5-4n-n^2}{4\left(n-3\right)\left(n+1\right)}$$

The last result implies that if $n \geq 3$ then (4.A.5) increases with p^c $\forall 1/2 < p^c < 1$, because $5-4n-n^2 < 0$.

If we set $p^c = 1/2$ then (4.A.5) simplifies to $\frac{n(n-1)}{2(n-3)(n+1)^2} > 0$.

Now we tackle the case $n = 2$. If $n = 2$, then $p^c = 1/\sqrt{3}$ and $\alpha = 1+\bar{x} = \left(1-\left(p^*\right)^2\right)/p^*$. After we plug the values of n, p^c and α, $n\phi_1\left(\bar{x},p^*,n\right)$ becomes the function of p^* only. More particularly,

$$\phi_1\left(\bar{x},p^*,2\right) = \frac{1-3\left(p^*\right)^2}{27\left(\left(p^*\right)^4-1\right)}\Big(-3+7\sqrt{3}p^*-9\left(p^*\right)^2$$
$$-8\sqrt{3}\left(p^*\right)^3+12\left(p^*\right)^4+3\sqrt{3}\left(p^*\right)^5\Big)$$

The fraction $\frac{1-3(p^*)^2}{27\left((p^*)^4-1\right)}$ is negative. The polynomial in the parenthesis is also negative because it does not have any real roots the interval [0, 1/2]. Thus, the polynomial is of the same sign for all $p^* \in [0, 1/2]$. If we set $p^* = 0$, then the expression equals $-3 < 0$. Therefore, $\phi_1(\bar{x}, p^*, 2) > 0$. ■

The derivative of the LHS of (4.13) with respect to $\widetilde{p}$. If we denote the LHS of (4.13) by $G(\widetilde{p})$ then

$$\frac{\partial G(\widetilde{p})}{\partial \widetilde{p}} = -\nu \frac{\partial G_2(\widetilde{p})}{\partial \widetilde{p}} - (1-\nu)\frac{\partial G_2(\widetilde{p})}{\partial \widetilde{p}}$$

where

$$\begin{aligned} G_1(\widetilde{p}) &= F(\widetilde{p})^n + \frac{1-F(\bar{x})^n}{1-F(\bar{x})}\widetilde{p} f(\bar{x}) - n\widetilde{p}\int_0^{\bar{x}-\widetilde{p}} F(\varepsilon+\widetilde{p})^{n-1} f'(\varepsilon+\widetilde{p})\, d\varepsilon \\ G_2(\widetilde{p}) &= F(\widetilde{p})^n + \frac{1-F(\bar{x}-p^c+\widetilde{p})^n}{1-F(\bar{x}-p^c+\widetilde{p})}\widetilde{p} f(\bar{x}-p^c+\widetilde{p}) \\ &\quad - n\widetilde{p}\int_0^{\bar{x}-p^c} F(\varepsilon+\widetilde{p})^{n-1} f'(\varepsilon+\widetilde{p})\, d\varepsilon \end{aligned}$$

The derivative of $G_1(\widetilde{p})$ with respect to $\widetilde{p}$ is identical to the derivative of the LHS of 4.6 with respect to p^* with the negative sign. Therefore, $\partial G_1(\widetilde{p}) / \partial \widetilde{p} > 0$.

$$\begin{aligned} \frac{\partial G_2(\widetilde{p})}{\partial \widetilde{p}} &= \frac{1-nF(z)^{n-1}+(n-1)F(z)^n}{(1-F(z))^2} f(z)^2 \widetilde{p} + \frac{1-F(z)^n}{1-F(z)}\left(\widetilde{p} f'(z) + f(z)\right) \\ &\quad - n\int_0^{\bar{x}-p^c} F(\varepsilon+\widetilde{p})^{n-1} f'(\varepsilon+\widetilde{p})\, d\varepsilon - n\widetilde{p}F(z)^{n-1} f'(z) + n\widetilde{p}F(\widetilde{p})^{n-1} f'(\widetilde{p}) \\ &\quad + nF(\widetilde{p})^{n-1} f(\widetilde{p}) \end{aligned}$$

where $z = \bar{x} - p^c + \widetilde{p}$.'

Now we apply integration by parts and get that

$$\begin{aligned} &n\int_0^{\bar{x}-p^c} F(\varepsilon+\widetilde{p})^{n-1} f'(\varepsilon+\widetilde{p})\, d\varepsilon \\ &= nF(z)^{n-1} f(z) - nF(\widetilde{p})^{n-1} f(\widetilde{p}) - n(n-1)\int_0^{\bar{x}-p^c} F(\varepsilon+\widetilde{p})^{n-2} f(\varepsilon+\widetilde{p})^2\, d\varepsilon \end{aligned}$$

We plug this value into the expression of $\frac{\partial G_2(\widetilde{p})}{\partial \widetilde{p}}$ and get that

$$\frac{\partial G_2(\widetilde{p})}{\partial \widetilde{p}} = \frac{1-nF(z)^{n-1}+(n-1)F(z)^n}{(1-F(z))^2}\widetilde{p}\left(f(z)^2 + (1-F(z))f'(z)\right)$$

$$+\frac{1-nF(z)^{n-1}+(n-1)F(z)^{n}}{1-F(z)}f(z)+nF(\widetilde{p})^{n-1}\left(2f(\widetilde{p})+f'(\widetilde{p})\,\widetilde{p}\right)$$
$$+n(n-1)\int_{0}^{\bar{x}-p^{c}}F(\varepsilon+\widetilde{p})^{n-2}f(\varepsilon+\widetilde{p})^{2}\,d\varepsilon>0 \qquad \blacksquare$$

Proof of claim 4.4. We denote the LHS of (4.17) by $H(p^*)$. This function decreases with p^*. Therefore, if we need to show that $\partial p^*/\partial\mu > 0$ then we need to show that the derivative of $H(p^*)$ with respect to μ is positive.

$$\begin{aligned}\frac{\mu^2}{p^*}\frac{\partial H(p^*)}{\partial\mu} &= \frac{1-F(\widetilde{x})^n}{1-F(\widetilde{x})}f(\widetilde{x})+nF\left(\frac{p^*}{\mu}\right)^{n-1}\left(f\left(\frac{p^*}{\mu}\right)+\frac{p^*}{\mu}f'\left(\frac{p^*}{\mu}\right)\right)\\ &-n\int_{p^*/\mu}^{\widetilde{x}}F(\varepsilon)^{n-1}f'(\varepsilon)\,d\varepsilon\\ &-\frac{1+(n-1)F(\widetilde{x})^n-nF(\widetilde{x})^{n-1}}{(1-F(\widetilde{x}))^2}\left(f(\widetilde{x})^2+f'(\widetilde{x})(1-F(\widetilde{x}))\right)\\ &\cdot\frac{\bar{s}/\mu}{(1-F(\widetilde{x}))}\end{aligned}$$

If $\widetilde{x}\to\bar{\varepsilon}$ (or $s\to 0$) then

$$\begin{aligned}\frac{\mu^2}{p^*}\frac{\partial H(p^*)}{\partial\mu} &= nf(\bar{\varepsilon})+nF\left(\frac{p^*}{\mu}\right)^{n-1}\left(f\left(\frac{p^*}{\mu}\right)+\frac{p^*}{\mu}f'\left(\frac{p^*}{\mu}\right)\right)\\ &-n\int_{p^*/\mu}^{\bar{\varepsilon}}F(\varepsilon)^{n-1}f'(\varepsilon)\,d\\ &=nf(\bar{\varepsilon})+nF\left(\frac{p^*}{\mu}\right)^{n-1}\left(f\left(\frac{p^*}{\mu}\right)+\frac{p^*}{\mu}f'\left(\frac{p^*}{\mu}\right)\right)-nf(\bar{\varepsilon})\\ &+nF\left(\frac{p^*}{\mu}\right)^{n-1}f\left(\frac{p^*}{\mu}\right)+n(n-1)\int_{p^*/\mu}^{\bar{\varepsilon}}F(\varepsilon)^{n-2}f(\varepsilon)^2\,d\varepsilon\\ &=nF\left(\frac{p^*}{\mu}\right)^{n-1}\left(2f\left(\frac{p^*}{\mu}\right)+\frac{p^*}{\mu}f'\left(\frac{p^*}{\mu}\right)\right)\\ &+n(n-1)\int_{p^*/\mu}^{\bar{\varepsilon}}F(\varepsilon)^{n-2}f(\varepsilon)^2\,d\varepsilon>0\end{aligned}$$

If $\varepsilon\sim U(0;1)$ then

$$\frac{\mu^2}{p^*}\frac{\partial H(p^*)}{\partial\mu}=\frac{1-\widetilde{x}^n}{1-\widetilde{x}}+n\left(\frac{p^*}{\mu}\right)^{n-1}-\frac{1+(n-1)\widetilde{x}^n-n\widetilde{x}^{n-1}}{(1-\widetilde{x})^2}\cdot\frac{\frac{1}{2}(1-\widetilde{x})^2}{(1-\widetilde{x})}$$

$$= n\left(\frac{p^*}{\mu}\right)^{n-1} + \frac{1+n\widetilde{x}^{n-1}-(n+1)\,\widetilde{x}^{n}}{2\,(1-\widetilde{x})} > 0$$

The inequality has been obtained because $1-\widetilde{x}>0$. Furthermore, if $\widetilde{x}=1$ then $1+n\widetilde{x}^{n-1}-(n+1)\,\widetilde{x}^{n}=0$; if $\widetilde{x}=0$ then $1+n\widetilde{x}^{n-1}-(n+1)\,\widetilde{x}^{n}=1>0$. At the extreme point of $1+n\widetilde{x}^{n-1}-(n+1)\,\widetilde{x}^{n}$ $\widetilde{x}$ equals $\widetilde{x}=(n-1)\,/\,(n+1)$. However,

$$1+n\left(\frac{n-1}{n+1}\right)^{n-1}-(n+1)\left(\frac{n-1}{n+1}\right)^{n}=1+\left(\frac{n-1}{n+1}\right)^{n-1}>0$$

Hence, $1+n\widetilde{x}^{n-1}-(n+1)\,\widetilde{x}^{n}\geq 0, \forall\widetilde{x}\in[0;1]$. $\blacksquare$.

Proof of claim 4.5. We start with the case $s\to 0$. The first order condition of a deviating firm for any $\mu\neq 1$ can be written as follows

$$\frac{1-F(\widetilde{x})^n}{1-F(\widetilde{x})}\left(1-F\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right)-\frac{p^d}{\mu}f\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right)\right)$$
$$+\,n\int_{\frac{p^d}{\mu}}^{\widetilde{x}-\frac{p^c}{\mu}+\frac{p^d}{\mu}}F\left(\varepsilon+\frac{p^c}{\mu}-\frac{p^d}{\mu}\right)^{n-1}\left(f(\varepsilon)+\frac{p^d}{\mu}f'(\varepsilon)\right)d\varepsilon=0$$

If we want to show that $\partial p^d/\partial\mu>0$ then we need to show that the LHS of the first order condition increases with μ. The LHS of the first order condition decreases with p^d. The proof is similar to the case $\mu=1$. Therefore, we omit it here.

Let us denote the LHS of the first order condition by $L\left(p^d\right)$. The derivative of $L\left(p^d\right)$ with respect to μ when $s\to 0$ encompass just the direct effect of μ and the effect via p^c because

$$\lim_{\widetilde{x}\to\bar{\varepsilon}}\frac{\partial\widetilde{x}}{\partial\mu}=\frac{\lim_{\widetilde{x}\to\bar{\varepsilon}}\int_{\widetilde{x}}^{\bar{\varepsilon}}(\varepsilon-\widetilde{x})\,dF(\varepsilon)}{\lim_{\widetilde{x}\to\bar{\varepsilon}}\mu\,(1-F(\widetilde{x}))\,/\,\mu}=0$$

We split the derivative of $L\left(p^d\right)$in two parts. The first part is the direct derivative of $L\left(p^d\right)$ with respect to μ. We denote this part by L_1. The second part is the effect of μ on $L\left(p^d\right)$ via p^c. We denote the derivative of $L\left(p^d\right)$ with respect to p^c by L_2. Thus the total derivative of $L\left(p^d\right)$ with respect to μ is $L_1+L_2\partial p^c/\partial\mu$.

$$L_1=\frac{1-F(\widetilde{x})^n}{1-F(\widetilde{x})}\left(-f\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right)\frac{p^c-2p^d}{\mu^2}-\frac{p^d\left(p^c-p^d\right)}{\mu^3}f'\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right)\right)$$

$$+ n \int_{p^d/\mu}^{\widetilde{x} - \frac{p^c - p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-2} \left(-(n-1)\frac{p^c - p^d}{\mu^2} f\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)\right.$$

$$\left.\cdot \left(f(\varepsilon) + \frac{p^d}{\mu} f'(\varepsilon)\right) - F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right) \frac{p^d}{\mu^2} f'(\varepsilon)\right) d\varepsilon$$

$$+ nF(\widetilde{x})^{n-1} \left(f\left(\widetilde{x} - \frac{p^c - p^d}{\mu}\right) + \frac{p^d}{\mu} f'\left(\widetilde{x} - \frac{p^c - p^d}{\mu}\right)\right) \frac{p^c - p^d}{\mu^2}$$

$$+ nF\left(\frac{p^c}{\mu}\right)^{n-1} \left(f\left(\frac{p^d}{\mu}\right) + \frac{p^d}{\mu} f'\left(\frac{p^d}{\mu}\right)\right) \frac{p^d}{\mu^2}$$

Then

$$\lim_{\widetilde{x} \to \bar{\varepsilon}} \frac{\mu^2}{n} L_1 = f\left(\bar{\varepsilon} - \frac{p^c - p^d}{\mu}\right) p^d - (n-1)$$

$$\cdot \int_{p^d/\mu}^{\bar{\varepsilon} - \frac{p^c - p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-2} f\left(\varepsilon + \frac{p^c - p^d}{\mu}\right) \left(p^c - p^d\right) \left(f(\varepsilon) + \frac{p^d}{\mu} f'(\varepsilon)\right) d\varepsilon$$

$$- \int_{p^d/\mu}^{\bar{\varepsilon} - \frac{p^c - p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-1} p^d f'(\varepsilon)\, d\varepsilon + F\left(\frac{p^c}{\mu}\right)^{n-1}$$

$$\cdot \left(f\left(\frac{p^d}{\mu}\right) + \frac{p^d}{\mu} f'\left(\frac{p^d}{\mu}\right)\right) p^d$$

Now we take the derivative of $L\left(p^d\right)$ with respect to p^c.

$$L_2 = \frac{1 - F(\widetilde{x})^n}{1 - F(\widetilde{x})} \left(f\left(\widetilde{x} - \frac{p^c - p^d}{\mu}\right) \frac{1}{\mu} + \frac{p^d}{\mu^2} f'\left(\widetilde{x} - \frac{p^c - p^d}{\mu}\right)\right)$$

$$+ \frac{n(n-1)}{\mu} \int_{p^d/\mu}^{\widetilde{x} - \frac{p^c - p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-2} f\left(\varepsilon + \frac{p^c - p^d}{\mu}\right) \left(f(\varepsilon) + \frac{p^d}{\mu} f'(\varepsilon)\right) d\varepsilon$$

$$- nF(\widetilde{x})^{n-1} \left(f\left(\widetilde{x} - \frac{p^c - p^d}{\mu}\right) + \frac{p^d}{\mu} f'\left(\widetilde{x} - \frac{p^c - p^d}{\mu}\right)\right) \frac{1}{\mu}$$

Then

$$\lim_{\widetilde{x} \to \bar{\varepsilon}} \frac{\mu^2}{n} L_2 \frac{\partial p^c}{\partial \mu}$$

$$= \int_{p^d/\mu}^{\bar{\varepsilon}-\frac{p^c-p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-2} f\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)(n-1)\, p^c \left(f(\varepsilon) + \frac{p^d}{\mu} f'(\varepsilon)\right) d\varepsilon$$

Consequently,

$$\lim_{\widetilde{x}\to\bar{\varepsilon}} \frac{\mu^2}{n} \frac{\partial L\left(p^d\right)}{\partial \mu} = f\left(\bar{\varepsilon} - \frac{p^c - p^d}{\mu}\right) p^d - \int_{p^d/\mu}^{\bar{\varepsilon}-\frac{p^c-p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-1} p^d f'(\varepsilon)\, d\varepsilon$$

$$+ p^d (n-1) \int_{p^d/\mu}^{\bar{\varepsilon}-\frac{p^c-p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-2} f\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)\left(f(\varepsilon) + \frac{p^d}{\mu} f'(\varepsilon)\right) d\varepsilon$$

$$+ F\left(\frac{p^c}{\mu}\right)^{n-1} \left(f\left(\frac{p^d}{\mu}\right) + \frac{p^d}{\mu} f'\left(\frac{p^d}{\mu}\right)\right) p^d$$

The second integral may be rewritten as

$$\int_{p^d/\mu}^{\bar{\varepsilon}-\frac{p^c-p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-1} p^d f'(\varepsilon)\, d\varepsilon$$

$$= p^d f\left(\bar{\varepsilon} - \frac{p^c - p^d}{\mu}\right) - p^d F\left(\frac{p^c}{\mu}\right)^{n-1} f\left(\frac{p^d}{\mu}\right)$$

$$- \int_{p^d/\mu}^{\bar{\varepsilon}-\frac{p^c-p^d}{\mu}} (n-1) F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-2} f\left(\varepsilon + \frac{p^c - p^d}{\mu}\right) p^d f(\varepsilon)\, d\varepsilon$$

Therefore, the last limit simplifies as follows

$$\lim_{\widetilde{x}\to\bar{\varepsilon}} \frac{\mu^2}{n} \frac{\partial L\left(p^d\right)}{\partial \mu} = F\left(\frac{p^c}{\mu}\right)^{n-1} \left(2f\left(\frac{p^d}{\mu}\right) + \frac{p^d}{\mu} f'\left(\frac{p^d}{\mu}\right)\right) p^d$$

$$+ p^d (n-1) \int_{p^d/\mu}^{\bar{\varepsilon}-\frac{p^c-p^d}{\mu}} F\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)^{n-2} f\left(\varepsilon + \frac{p^c - p^d}{\mu}\right)$$

$$\cdot \left(2f(\varepsilon) + \frac{p^d}{\mu} f'(\varepsilon)\right) d\varepsilon > 0$$

If $\varepsilon \sim U(0, 1)$then the first order condition of a deviating firm simplifies to

$$1 + \frac{1 - \widetilde{x}^n}{1 - \widetilde{x}} \left(\frac{p^c}{\mu} - \frac{2p^d}{\mu}\right) - \left(\frac{p^c}{\mu}\right)^n = 0$$

As a result we get that

$$p^d = \frac{1}{2}\left(p^c + \frac{(1-\widetilde{x})\,\mu}{1-\widetilde{x}^n}\left(1-\left(\frac{p^c}{\mu}\right)^n\right)\right)$$

Note that $p^c/\mu = (n+1)^{1/n}$ if $\varepsilon \sim U(1,0)$. Therefore,

$$\begin{aligned}2\frac{n+1}{n}\frac{\partial p^d}{\partial \mu} &= \frac{1-\widetilde{x}}{1-\widetilde{x}^n} + \frac{1}{n}\left(\frac{p^c}{\mu}\right)^{1-n} - \frac{1-\widetilde{x}}{2}\frac{1+(n-1)\,\widetilde{x}^n - n\widetilde{x}^{n-1}}{(1-\widetilde{x}^n)^2} \\ &= \frac{1-\widetilde{x}}{2\,(1-\widetilde{x}^n)^2}\left(1+n\widetilde{x}^{n-1}-(n+1)\,\widetilde{x}^n\right) + \frac{1}{n}\left(\frac{p^c}{\mu}\right)^{1-n} > 0 \qquad \blacksquare\end{aligned}$$

Proof of claim 4.6. The profit of a deviant may be written as follows

$$\begin{aligned}\frac{1}{p^d}\pi^d &= \frac{1}{n}\frac{1-F(\widetilde{x})^n}{1-F(\widetilde{x})}\left(1-F\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right)\right) \\ &\quad + \int_{\frac{p^d}{\mu}}^{\widetilde{x}-\frac{p^c}{\mu}+\frac{p^d}{\mu}} F\left(\varepsilon+\frac{p^c}{\mu}-\frac{p^d}{\mu}\right)^{n-1} f(\varepsilon)\,d\varepsilon\end{aligned}$$

We start with the case when $s \to 0$. We split the derivative of $\lim_{\widetilde{x}\to\bar{\varepsilon}} \pi^d$ in two parts. The first part will be the direct effect of μ on π^d. This part we denote by P_1. The second effect of μ is indirect and comes via p^c. This part of the derivative we label P_2. The effect of μ via $\widetilde{x}$ vanishes if $s \to 0$, ad the derivative with respect to p^d equals zero.

$$\begin{aligned}\frac{1}{p^d}P_1 = &-\frac{1-F(\widetilde{x})^n}{n\,(1-F(\widetilde{x}))} f\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right)\frac{p^c-p^d}{\mu^2} + F(\widetilde{x})^{n-1} f\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right) \\ &\cdot \frac{p^c-p^d}{\mu^2} + F\left(\frac{p^c}{\mu}\right)^{n-1} f\left(\frac{p^d}{\mu}\right)\frac{p^d}{\mu^2} \\ &-(n-1)\int_{p^d/\mu}^{\widetilde{x}-\frac{p^c-p^d}{\mu}} F\left(\varepsilon+\frac{p^c-p^d}{\mu}\right)^{n-2} f\left(\varepsilon+\frac{p^c-p^d}{\mu}\right) f(\varepsilon)\,\frac{p^c-p^d}{\mu^2}d\varepsilon\end{aligned}$$

Then

$$\frac{\mu^2}{p^d}\lim_{s\to 0} P_1 = F\left(\frac{p^c}{\mu}\right)^{n-1} f\left(\frac{p^d}{\mu}\right) p^d$$

$$-\left(p^c-p^d\right)(n-1)\int_{\frac{p^d}{\mu}}^{\bar{\varepsilon}-\frac{p^c}{\mu}+\frac{p^d}{\mu}} F\left(\varepsilon+\frac{p^c-p^d}{\mu}\right)^{n-2} f\left(\varepsilon+\frac{p^c-p^d}{\mu}\right) f(\varepsilon)\,d\varepsilon$$

$$\begin{aligned}\frac{1}{p^d}P_2 &= \frac{1-F(\widetilde{x})^n}{n(1-F(\widetilde{x}))} f\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right)\frac{1}{\mu}\frac{p^c}{\mu} - F(\widetilde{x})^{n-1} f\left(\widetilde{x}-\frac{p^c-p^d}{\mu}\right)\frac{p^c}{\mu^2}\\ &\quad + (n-1)\int_{p^d/\mu}^{\widetilde{x}-\frac{p^c-p^d}{\mu}} F\left(\varepsilon+\frac{p^c-p^d}{\mu}\right)^{n-2} f\left(\varepsilon+\frac{p^c-p^d}{\mu}\right) f(\varepsilon)\frac{p^c}{\mu^2}d\varepsilon\end{aligned}$$

If the search cost goes to zero then P_2 simplifies to

$$\frac{\mu^2}{p^d}\lim_{s\to 0}P_2 = (n-1)\,p^c\int_{p^d/\mu}^{\bar{\varepsilon}-\frac{p^c-p^d}{\mu}} F\left(\varepsilon+\frac{p^c-p^d}{\mu}\right)^{n-2} f\left(\varepsilon+\frac{p^c-p^d}{\mu}\right) f(\varepsilon)\,d\varepsilon$$

Hence,

$$\begin{aligned}&\frac{\mu^2}{p^d}\lim_{s\to 0}\frac{\partial\pi^d}{\partial p^d} = F\left(\frac{p^c}{\mu}\right)^{n-1} f\left(\frac{p^d}{\mu}\right)p^d\\ &+(n-1)\,p^d\int_{\frac{p^d}{\mu}}^{\bar{\varepsilon}-\frac{p^c}{\mu}+\frac{p^d}{\mu}} F\left(\varepsilon+\frac{p^c-p^d}{\mu}\right)^{n-2} f\left(\varepsilon+\frac{p^c-p^d}{\mu}\right) f(\varepsilon)\,d\varepsilon>0\end{aligned}$$

If $\varepsilon\sim U(0,1)$. Then the profit expression simplifies to

$$n\pi^d = \frac{\left(p^d\right)^2}{\mu}\frac{1-\widetilde{x}^n}{1-\widetilde{x}}$$

Then

$$\begin{aligned}\frac{\partial n\pi^d}{\partial\mu} &= \frac{\left(p^d\right)^2}{\mu}\frac{1+(n-1)\widetilde{x}^n-n\widetilde{x}^{n-1}}{(1-\widetilde{x})^2}\frac{\frac{1}{2}(1-\widetilde{x})^2}{\mu(1-\widetilde{x})} - \frac{\left(p^d\right)^2}{\mu^2}\frac{1-\widetilde{x}^n}{1-\widetilde{x}}\\ &\quad+\frac{p^d}{\mu}\frac{1+n\widetilde{x}^{n-1}-(n+1)\widetilde{x}^n}{2(1-\widetilde{x}^n)}\frac{n}{n+1}+\frac{p^d}{\mu}\frac{1-\widetilde{x}^n}{1-\widetilde{x}}\frac{p^c}{\mu}>0 \qquad \blacksquare\end{aligned}$$

Proof of proposition 4.2. The sign of $\partial\widehat{\delta}/\partial\mu$ is the same as the sign of $\partial\gamma/\partial\mu$ where

$$\gamma=\frac{\pi^d-\pi^c}{\pi^c-\pi^*}=\frac{\widehat{\delta}}{1-\widehat{\delta}}$$

and

$$\operatorname{sgn}\left[\frac{\partial\gamma}{\partial\mu}\right]=\operatorname{sgn}\left[\frac{\partial\left(\pi^{d}-\pi^{c}\right)}{\partial\mu}\left(\pi^{c}-\pi^{*}\right)-\frac{\partial\left(\pi^{c}-\pi^{*}\right)}{\partial\mu}\left(\pi^{d}-\pi^{c}\right)\right]\tag{4.A.6}$$

(A) The case when $\widetilde{x}\to 1$ (or $s\to 0$). We start by deriving every element of the RHS of (4.A.6)

$$\frac{\partial\pi^{c}}{\partial\mu}=\left(\frac{p^{c}}{\mu}\right)^{n+1}$$

$$\lim_{\widetilde{x}\to 1}\frac{\partial\left(\pi^{d}-\pi^{c}\right)}{\partial\mu}=\frac{1}{4}\left(\frac{p^{c}}{\mu}+\left(\frac{p^{c}}{\mu}\right)^{n}\right)^{2}-\left(\frac{p^{c}}{\mu}\right)^{n+1}=\frac{1}{4}\left(\frac{p^{c}}{\mu}-\left(\frac{p^{c}}{\mu}\right)^{n}\right)^{2}$$

$$\lim_{\widetilde{x}\to 1}\left(\pi^{c}-\pi^{*}\right)=p^{c}\left(\frac{p^{c}}{\mu}\right)^{n}-\frac{\left(p^{*}\right)^{2}}{\mu}$$

$$\lim_{\bar{x}\to 1}\left(\pi^{d}-\pi^{c}\right)=\frac{\mu}{4}\left(\frac{p^{c}}{\mu}+\left(\frac{p^{c}}{\mu}\right)^{n}\right)^{2}-p^{c}\left(\frac{p^{c}}{\mu}\right)^{n}=\frac{\mu}{4}\left(\frac{p^{c}}{\mu}-\left(\frac{p^{c}}{\mu}\right)^{n}\right)^{2}$$

The derivative of π^{*} with respect to μ is as follows:

$$\begin{aligned}n\frac{\partial\pi^{*}}{\partial\mu}&=-\frac{\left(p^{*}\right)^{2}}{\mu^{2}}\frac{1-\widetilde{x}^{n}}{1-\widetilde{x}}-\frac{2p^{*}}{\mu}\frac{\partial H\left(p^{*}\right)/\partial\mu}{\partial H\left(p^{*}\right)/\partial p^{*}}\frac{1-\widetilde{x}^{n}}{1-\widetilde{x}}\\&\quad+\frac{\left(p^{*}\right)^{2}}{\mu}\frac{1-n\widetilde{x}^{n-1}+(n-1)\widetilde{x}^{n}}{(1-\widetilde{x})^{2}}\frac{\partial\widetilde{x}}{\partial\mu}\end{aligned}$$

$$\lim_{\bar{x}\to 1}\frac{\partial\left(\pi^{c}-\pi^{*}\right)}{\partial\mu}=\left(\frac{p^{c}}{\mu}\right)^{n+1}-\left(\frac{p^{*}}{\mu}\right)^{2}$$

Then

$$\begin{aligned}&\lim_{\bar{x}\to 1}\left(\frac{\partial\left(\pi^{d}-\pi^{c}\right)}{\partial\mu}\left(\pi^{c}-\pi^{*}\right)-\frac{\partial\left(\pi^{c}-\pi^{*}\right)}{\partial\mu}\left(\pi^{d}-\pi^{c}\right)\right)\\&=\mu\left[\frac{1}{4}\left(\frac{p^{c}}{\mu}-\left(\frac{p^{c}}{\mu}\right)^{n}\right)^{2}\right]\left[\left(\frac{p^{c}}{\mu}\right)^{n+1}-\left(\frac{p^{*}}{\mu}\right)^{2}\right]\end{aligned}$$

$$-\mu\left[\left(\frac{p^c}{\mu}\right)^{n+1}-\left(\frac{p^*}{\mu}\right)^2\right]\left[\frac{1}{4}\left(\frac{p^c}{\mu}-\left(\frac{p^c}{\mu}\right)^n\right)^2\right]=0$$

(B) Now let us turn to the high search cost, i.e. $\widetilde{x}\to\frac{p^c}{\mu}$. Before we write down the expression of the derivative of γ we introduce a new notation. Let us define $\eta\equiv\frac{p^c}{\mu}$ and $\theta=\frac{p^*}{\mu}$. These two new variables do not depend on μ because $\eta=(n+1)^{-1/n}$ and θ is defined by equation

$$1-\frac{1-\eta^n}{1-\eta}\theta-\theta^n=0$$

.

As a result, both η and θ are the functions of n only.

We again start with each element of (4.A.6) separately. The derivative of π^c with respect to μ remains the same as before, i.e. $\partial\pi^c/\partial\mu=\eta^{n+1}$ and $\pi^c=p^c\eta^n=\mu\eta^{n+1}$ The deviation price approaches to $\mu/2$ if $\widetilde{x}\to\eta$. Then the expression of the deviation profit and its derivative with respect to μ simplify as follows

$$\lim_{s\to\bar{s}_n}\pi^d=\frac{1}{\mu}\frac{\mu^2}{4}\frac{1-\eta^n}{1-\eta}\frac{1}{n}=\frac{\mu}{4}\frac{\eta^n}{(1-\eta)}$$

$$\begin{aligned}\lim_{s\to\bar{s}_n}\frac{\partial\pi^d}{\partial\mu}&=\frac{\mu}{4}\frac{1+(n-1)\eta^n-n\eta^{n-1}}{2n\mu(1-\eta)}-\frac{1}{4}\frac{1-\eta^n}{n(1-\eta)}\\&+\frac{1}{2}\frac{1+n\eta^{n-1}-(n+1)\eta^n}{2(1-\eta^n)}\frac{1}{n+1}+\frac{1}{2}\eta\frac{1-\eta^n}{1-\eta}\frac{1}{n}\\&=\frac{1+(n-1)\eta^n-n\eta^{n-1}}{8n(1-\eta)}-\frac{1}{4(n+1)(1-\eta)}+\frac{\eta^{n-1}}{4}\\&+\frac{\eta}{2(1-\eta)(n+1)}=\frac{-4\eta^2-1+2\eta}{8(n+1)(\eta-1)\eta}\end{aligned}$$

The expression of competitive profits and its derivative with respect to μ is the function of $\eta(n)$, $\theta(n)$, n and μ:

$$\lim_{s\to\bar{s}_n}\pi^*=\frac{(p^*)^2}{\mu}\frac{1-\eta^n}{n(1-\eta)}=\frac{\theta^2\mu\eta^n}{1-\eta}$$

$$\lim_{s\to\bar{s}_n}\frac{\partial p^*}{\partial\mu}=\frac{n\frac{\theta}{\mu}\theta^{n-1}+\frac{\theta}{\mu}\frac{1+n\eta^{n-1}-(n+1)\eta^n}{2(1-\eta)}}{\frac{n\eta^n}{1-\eta}\frac{1}{\mu}+n\theta^{n-1}\frac{1}{\mu}}=\frac{\theta^n+\theta\frac{\eta^{n-1}}{2(1-\eta)}}{\frac{\eta^n}{1-\eta}+\theta^{n-1}}$$

$$= \frac{\theta\left(2\theta^{n-1}\left(1-\eta\right)+\eta^{n-1}\right)}{2\eta^{n}+2\theta^{n-1}\left(1-\eta\right)}$$

$$\begin{aligned}\lim_{s\to\bar{s}_n}\frac{\partial\pi^*}{\partial\mu} &= -\frac{\theta^2\eta^n}{1-\eta}+\frac{2\theta\eta^n}{1-\eta}\frac{\theta\left(2\theta^{n-1}\left(1-\eta\right)+\eta^{n-1}\right)}{2\eta^{n}+2\theta^{n-1}\left(1-\eta\right)}\\ &+\frac{\theta^2}{n}\frac{1-n\eta^{n-1}+\left(n-1\right)\eta^n}{2\left(1-\eta\right)}\\ &= \frac{\theta^2\left(4\eta\theta^n-4\eta^2\theta^n+(1+n)\eta^{1+n}\theta^n+\eta^n\left(\theta-(1+n)\theta^n\right)\right)}{2(-1+\eta)\eta\left(-\theta+(1+n)(-1+\eta)\theta^n\right)}\\ &= \frac{\theta^2\left(\theta^n\left(1-\eta\right)\left(4\eta-1\right)+\eta^n\theta\right)}{2(1-\eta)\eta\left(\theta+(1+n)(1-\eta)\theta^n\right)}\end{aligned}$$

The product heterogeneity parameter μ enters the RHS of (4.A.6) linearly because it enters the differences in profits only. Moreover, $\mu \geq 0$. Therefore, the sign of the derivative of γ depends only on n when $\widetilde{x} \to \eta$. We divide the RHS of (4.A.6) by μ and plot the expression for different values of n (see Figure 4.A.1). The expression is positive. Therefore $\lim_{s\to\bar{s}_n}\partial\widehat{\delta}/\partial\mu > 0$.

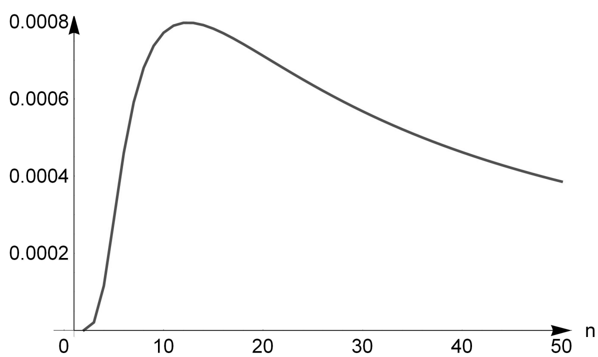


Figure 4.A.1. The value of (4.A.6) when $\widetilde{x} \to \eta$

Proof of proposition 4.3. We use the notation from the proof of proposition 4.2 here, i.e. $p^c/\mu = \eta$, $p^*/\mu = \theta$ and we denote p^d/μ by ξ. Then the first order condition of a firm in a competitive market can be rewritten as

$$1-\frac{1-F\left(\widetilde{x}\right)^n}{1-F\left(\widetilde{x}\right)}f\left(\widetilde{x}\right)\theta-F\left(\theta\right)^n+n\theta\int_{\theta}^{\widetilde{x}}F\left(\varepsilon\right)^{n-1}f'\left(\varepsilon\right)d\varepsilon=0$$

Then the derivative of θ with respect to $\widetilde{x}$ looks like the same as the derivative of p^* with respect to $\bar{x}$, given that $\mu = 1$. Furthermore, the restrictions on θ in terms of $\widetilde{x}$

and n are the same as the restriction on p^* in terms of n and $\bar{x}$.

If we replace p^c/μ by η in (4.19) then we get similar equation to (4.10) with η instead of p^c and all restrictions that are imposed on p^c by (4.10) are the same for η from equation (4.19).

If we replace p^d/μ with ξ in the first order condition of a deviant, then we get the following expression

$$\begin{aligned}&\frac{1-F\left(\widetilde{x}\right)^n}{1-F\left(\widetilde{x}\right)}\left(1-F\left(\widetilde{x}-\eta+\xi\right)-\xi f\left(\widetilde{x}-\eta+\xi\right)\right)\\&+n\int_0^{\widetilde{x}-\eta}F\left(\varepsilon+\eta\right)^{n-1}\left(f\left(\varepsilon+\xi\right)+\xi f'\left(\varepsilon+\xi\right)\right)d\varepsilon=0\end{aligned}\tag{4.A.7}$$

Thus, the derivative of ξ with respect to $\widetilde{x}$ looks almost the same as the derivative of p^d with respect to $\bar{x}$, given that $\mu = 1$. Moreover, the restrictions on the value of ξ in terms of $\widetilde{x}$ and n from equation (4.A.7) are the same as the restrictions on the value of p^d in terms of $\bar{x}$ and n from equation (4.11).

We rewrite the competitive profit as $\pi^* = \frac{\mu}{n}\theta\left(1-F\left(\theta\right)^n\right)$. The profit of a cartel member, given that $\mu \neq 1$, can be written as $\pi^c = \frac{\mu}{n}\eta\left(1-F\left(\eta\right)^n\right)$. Similarly,

$$\pi^d=\frac{\mu}{n}\xi\left(\frac{1-F\left(\widetilde{x}\right)^n}{1-F\left(\widetilde{x}\right)}\left(1-F\left(\widetilde{x}-\eta+\xi\right)\right)+n\int_0^{\widetilde{x}-\eta}F\left(\varepsilon+\eta\right)^{n-1}f\left(\varepsilon+\xi\right)d\varepsilon\right)$$

If we replaced the new parameters with $\bar{x}$, p^c, p^* and p^d in the expression of the profits we would get exactly the same expressions of the profits that we have derived for the case $\mu = 1$, except factor μ in the front.

The same holds for the derivatives of the profits. For instance, the derivative of the competitive profit with respect to $\widetilde{x}$ may be written as follows

$$\frac{\partial\pi^*}{\partial\widetilde{x}}=\frac{\mu}{n}\left(1-F\left(\theta\right)^n-nF\left(\theta\right)^{n-1}f\left(\theta\right)\theta\right)\frac{\partial\theta}{\partial\widetilde{x}}$$

Similarly,

$$\begin{aligned}\frac{\partial\pi^d}{\partial\widetilde{x}}=\frac{\mu}{n}\xi&\left(\frac{1+(n-1)F\left(\widetilde{x}\right)^n-nF\left(\widetilde{x}\right)^{n-1}}{\left(1-F\left(\widetilde{x}\right)\right)}\left(1-F\left(\widetilde{x}-\eta+\xi\right)\right)\right.\\&\left.\cdot\left(\frac{f\left(\widetilde{x}\right)}{1-F\left(\widetilde{x}\right)}-\frac{f\left(\widetilde{x}-\eta+\xi\right)}{1-F\left(\widetilde{x}-\eta+\xi\right)}\right)\right)\end{aligned}$$

If we write down the derivative of $\widehat{\delta}$ with respect ro $\widetilde{x}$ and divide it by μ^2, then the sign of the new expression would be the same as the sign of $\partial\widehat{\delta}/\partial\widetilde{x}$. Then, if we replace $\widetilde{x}$, η, ξ, θ with $\bar{x}$, p^c, p^* and p^d in the new expression then we get the derivative of the critical discount factor $\widehat{\delta}$ with respect to $\bar{x}$. Consequently, the sign of $\partial\widehat{\delta}/\partial\widetilde{x}$ is the same as the sign of $\partial\widehat{\delta}/\partial\bar{x}$. ■

Chapter 5

Deposit demand estimation under costly consumer search

5.1 Introduction

Concentration in the market for banking services varies across the European Union (the EU) a lot. For instance, in the period 2005-2009 the Herfindahl-Hirshman Index (HHI) for the banking market was relatively low in Germany (~ 200), and it fluctuated in the interval between 400 and 500 in the United Kingdom. Meanwhile, the index was over 2500 in Finland, around 2000 in the Netherlands and exceeded 3000 in Estonia in the same period.[1] Competition authorities pay attention to high HHI values, and any potential merger or acquisition goes under scrutiny if the concentration index is already relatively high in the pre-merger market.

Banks are engaged in many different economic activities. They are the payment intermediaries between firms and their customers; banks accept deposits and issue loans, mortgages, exchange currencies, perform some investment activities, etc. The clientele of a bank is very heterogeneous in terms of demanded services. For instance, an individual may prefer to get a package with a current account, an internet banking option, a credit card and a mortgage. However, a firm probably prefers to relate an overdraft option to its current account and looks for transaction services with the lowest transaction price. Therefore, it is difficult to judge the market power of banks precisely just from general banking market concentration measures.

[1] For more details see European Central Bank (2010)

In this chapter we provide a method to estimate saving deposit demand of households under the assumption that depositors incur positive search costs in order to find the best saving deposit contracts. The estimated parameters of the demand functions have a strong impact on the assessment of market power. Therefore, the demand function specification and its estimation results are important in counterfactual merger simulations. Our model is not estimated with actual data because the necessary data are not available at this moment. Though, we provide simulation results where we show that the estimated model parameters are biased if the search costs are ignored or if the search costs are modeled as a contract's characteristic instead of as a cost that limits a consumer's consideration set.

The average consumer consideration set is smaller when she searches sequentially at positive costs than when there are no search costs. The assumption that the search costs are zero or the inclusion of the search costs directly in the utility function implies that a consumer compares all the existing contracts, which is different from the true data generating process. Therefore, the estimated parameters are biased.

The concept *deposit demand* may sound slightly misleading because of the following reason. A bank invests money that is deposited by its customers, pays the money out when required and makes loans at interest. The institution makes its profit by charging higher interest rates on loans than it pays out to the depositors. Thus, if we treat a bank as an ordinary firm then a deposit is a production input and the interests on the deposits are variable production costs. Hence, the relationship between an offered interest rate and the sum of accepted deposits should be called *deposit supply* from the point of view of a bank. However, it is usually assumed that a bank supplies its customers with services and a saving deposit is a service of a bank. Therefore, we use the term *deposit demand* instead of *deposit supply* in this chapter.

We use the multinomial logit (MNL) model to derive and estimate the saving deposit demand. The application of this discrete choice modeling technique is not a novelty in the analysis of the retail banking sector. Dick (2008) and Ho and Ishii (2011) used a random coefficient MNL for the estimation of saving deposit demand in the US. Zhou (2008), Nakane et al. (2006) and Molnár et al. (2007) used a mixed MNL to estimate structural banking market models in the US, Brasil and Hungary respectively. Knittel and Stango (2008) and Ishii (2005) used this approach to analyze how the degree of interbank ATM network compatibility affects consumer choices for saving deposits and banks' incentives to invest. Our model differs from the mentioned ones by assuming costly consumer search in the saving deposit market. It is assumed in all the above mentioned papers that bank customers are perfectly informed about the saving deposit contracts available in the market. On the contrary, we assume that a typical bank customer does not know the precise utilities of all the saving contracts. In our model bank customers get the information about interest rates and some characteristics of banks for free. However, a customer needs to spend some time to learn the rest of contract and bank characteristics, e.g. friendliness of the staff, the duration of a financial month, etc. at some positive costs. The modeling assumptions are based on the surveys and reviews of the banking market in European countries.

Knowledge about consumers' consideration sets is very important for the identification of search costs. Sometimes the data suggest what alternatives were observed and compared by a single consumer before the purchase decision was made. For instance, Honka (2010) had information about the observed and chosen offers of every consumer when she estimated an auto insurance demand model with non-sequential consumer search. Kim et al. (2010) inferred the sequential search order by using *Amazon.com* view rank data in the estimation of the online demand for camcorders. Furthermore, Koulayev (2010) and Brynjolfsson et al. (2010) had information about hotels and books respectively that were observed by consumers. De los Santos et al. (2011) had data on the browsing history of every consumer when they analysed whether sequential or non-sequential search models were in line with the behaviour of consumers buying books online.

If the data does not suggest the size and the composition of consumers' consideration sets then the insights from economic theory can be used to judge which alternatives should have higher probabilities to be considered. The characteristics of products (quality, prices, etc.) affect the list of alternatives compared by consumers. If products are homogeneous, the price is all that matters for consumers. Then the data on prices is sufficient to determine the order in which alternatives get into a consumer's consideration set. Furthermore, the size of the search cost determines the size of the consideration set. If the search costs are high then a consumer samples very little offers. Hence, the most expensive offers are accepted by the consumers with the highest search costs and the low search cost consumers pay lower prices because they sample more items. Hong and Shum (2006) used this fact when they discussed the identification of the search cost distribution with sequential and non-sequential consumer search in a homogeneous product market.

Product heterogeneity makes the ranking of alternatives complicated because there are factors other than the price that affect consumer choice. Hortaçsu and Syverson (2004) used the fact that alternatives are of vertically differentiated when they estimated a search cost distribution in the market for S&P 500 funds. They needed information both on prices and market shares of the funds for the identification of the search cost distribution. Wildenbeest (2011) estimated a search cost distribution with vertically differentiated products without the information on market shares. However, he focused on mixed strategy equilibrium in utility levels, whereas the market equilibrium was in pure strategies in the model of Hortaçsu and Syverson (2004).

If products are horizontally differentiated then it is not clear how the products should be ranked. Then the number of possible consideration sets increases very rapidly with the number of alternatives and complicates the estimation process. Moraga-González et al. (2011) address this problem when they estimate a market model for cars. In their model consumers search the car dealers non-sequentially and there is no clear ranking of alternatives due to horizontal product differentiation. They have overcome the computationally intensive many-consideration set problem by using importance sampling.

Our set-up is similar to the one of Moraga-González et al. (2011) because neither the search behaviour of bank customers nor their consideration sets are observed. Furthermore, the saving contracts are treated as horizontally differentiated products in the demand specification. Therefore, no criterion can be used for the ranking of credit institutions and contracts. We build our model on the theoretical sequential costly consumer search framework first proposed by Wolinsky (1986) and further analyzed by Anderson and Renault (1999), Armstrong et al. (2009) and Zhou (2011). Hence, bank customers search the banks sequentially with perfect recall. A customer observes all saving contracts of a bank after she pays the search cost. Any search order may happen with positive probability, and the number of search orders increases very fast with the number of existing banks. Therefore, the estimation of the demand may be computationally difficult if there are many banks in the market and a customer is free to visit any number of credit institutions. In order to lower the computational burden, we impose some restrictions on the search behaviour of bank customers in model simulation.

The remainder of this chapter is organized as follows. We describe the modelling assumptions in section 5.2. The search and choice behaviour of bank customers is described in section 5.3.1. The derivation of market shares in the saving contract market is given in section 5.3.2. The simulation results are in section 5.4.

Finally, possible future model development is discussed in section 5.5.

5.2 Modeling assumptions

We use the following list of assumptions to derive the sequential search model for the saving deposits.

A1 Bank customers observe a few saving deposit contracts' characteristics for free and have to search for the rest of the characteristics at positive costs.

A2 Bank customers search banks sequentially with perfect recall and the offers of the first bank are learnt for free.

A3 The search cost per bank equals the sum of a constant c and a customer and bank specific term ν that is distributed according to a continuous differentiable distribution function $H(\nu)$.

A4 Bank customers perceive fixed term saving deposit contracts as differentiated products.

A5 A customer puts only a part of its assets (an emergency fund) under a saving deposit contract and does not sign more than one saving contract.

A6 A bank customer does not consider switching costs before signing a saving deposit contract. Furthermore, banks do not affect the choice of a customer by bundling and tying their offered services.

A bank customer wants to put her savings in the bank that offers the highest utility. She observes the offered interest rates, and some characteristics of a bank at no costs. However, some factors can be observed only after paying a search cost. For instance, Paswan, Spears, Hasty, and Ganesh (2004) show that bank customers pay attention to the knowledge and experience of employees, empathy, friendliness, a feeling of confidence in a bank, etc. These characteristics of banks are customer specific most of the time. Therefore, we assume that a consumer must spend some efforts or have some monetary expenditures in order to learn this kind of information.

A bank customer starts searching from the bank where she has her current account or handles all her daily transactions because it does not cost anything to search there. This assumption is made because bank customers have much experience about the services in the banks where they have their current accounts. Moreover, a customer gets some additional information about the bank's saving offers while she handles her credit card, internet banking, etc. issues. If a customer is not satisfied with the best offer in the bank where she has her current account then she searches other credit institutions. She searches banks sequentially with perfect recall.

The search cost per bank may be determined in many ways. For instance, it has been shown by Devlin (2002) that the location and the reputation of a bank are very important choice criteria for a bank customer in the United Kingdom. Relatively many surveyees stressed the importance of the bank location in the survey of Lee and Marlowe (2003). Additionally, Bexley (2005) find that recommendations from existing bank customers are very important choice criteria for choosing a financial institution. Therefore, the search costs per bank could be defined as a function of bank characteristics: distance, market share in previous periods, etc. In order to make the derivation of market shares easier we assume that the search costs per bank equal the sum of constant c plus bank and customer specific error term ν. The random term allows us to deal with the search cost heterogeneity among customers, which happens because bank customers vary in their disutility that they experience when they try to reach a credit institution (Boyd, Myron, and White, 1994; Lee and Marlowe, 2003; Paswan et al., 2004).

Devlin and Gerrard (2005, 2004) have analysed bank customer survey data that was collected in Great Britain in 2000. They find that bank customers consider the location and the image of a bank, home banking options, opening hours, interest rates on savings and credit and other charges before they choose where to open their current accounts. Furthermore, the duration of a financial year (month), the conditions to break a non-expired fixed term saving deposit contract, etc. varies among banks. Hence, bank customers consider saving deposit contracts as differentiated products.

An individual may earn interest on her savings in several ways. For instance, she may sign a saving contract, buy some shares and (or) bonds, engage in the trade of financial derivatives, etc. We assume that a bank customer makes rational investment decisions. Saving deposits have a small return compared to other types of investment. However, a saving deposit is a lower risk and more liquid investment than the investment in shares, capital funds, financial derivatives, etc. Therefore, it is advised for households to keep their emergency funds as saving deposits, and the rest of their money to invested elsewhere.[2]

[2] The purpose of an emergency fund is to cover the necessary expenses of an individual (household) if there is a sudden decrease in income or an unexpected increase in expenditures. Usually it is stated that the size of an emergency fund must be between three to six months of current monthly expenditures.

It is obligatory for commercial banks to follow national deposit insurance schemes in most European countries. The insured deposit size exceeds the recommended size of an emergency fund on average in the EU.[3] Furthermore, banks are very vulnerable to the behaviour of their customers. The collapse of one bank makes the customers of other banks more suspicious and may cause the bankruptcy of other credit institutions. Hence, spreading of an emergency fund across several banks does not necessarily help to avoid the case that the whole emergency fund needs to be recovered from the saving deposit insurance. Moreover, it takes more efforts from a bank customer to monitor her investment in several banks compared to the case when only one saving deposit contract is signed.

Tying, bundling and consumer switching costs are considered to be important aspects of the retail banking market in Europe (see the EC, 2007; the EC, 2009). However, the switching costs and bundling do not have a strong effect on consumer choice where to sign a saving deposit contract. Bank customers had to name the reasons why they signed their saving contracts with particular banks in the EC survey in 2009. According to the survey results, only 5.83% bank customers signed their saving contracts with particular banks just because they had their current accounts there. Furthermore, it is acknowledged that in some countries, e.g. the Netherlands, consumers pay relatively low costs for their current accounts and have multiple accounts.[4] If a customer has current accounts in several banks then she can transfer her money from one credit institution to another at no additional costs. Additionally, it has been stated in the OECD and the EC reports that in several European countries banks do not take advantage via tying arrangements, or the tying practice that is related to current accounts is mostly exercised by issuing loans and selling insurance products. Therefore, we abstract from switching costs in our model.

Estimation of a demand model raises the issue of endogeneity because the contract interest rate may be correlated with the unobserved characteristics of a contract. Therefore, instruments are necessary to estimate the model. The set of instruments that are used in the empirical banking market models with the unobserved product characteristics varies a lot. For instance, Nakane et al. (2006) include per-

However, the exact size of an emergency fund depends on the social security system of a particular country and the individual's abilities to tackle the unexpected gap between her income and expenditures.

[3] It is stated in the directive of the EU No. 2009/14/EC Art. 1.3 that "*by 31 December 2010, Member States shall ensure that the coverage for the aggregate deposits of each depositor shall be set at EUR 100 000 in the event of deposits being unavailable*". Meanwhile, the average yearly disposable income per person did not exceeded EUR 23 000 in the EU in 2007 (EUROSTAT).

[4] Boot (2007), OECD (2007)

sonnel and operational costs, credit risk, liquidity coefficients, the ratio of net worth to operational assets, the ratio of loans to operational assets next to BLP instruments (characteristics of the rivals in each market) in their model. A similar approach was applied by Ho and Ishii (2011), Nakane et al. (2006) and Dick (2008), who used BLP instruments in addition to a list of cost shifters such as the expenses on premises and equipment, credit risk cost variable, depreciation, etc. Zhou (2008) and Knittel and Stango (2008) used only BLP instruments in their models.

We provide two suggestions for the instruments here. First of all, if the interbank lending market is very liquid and a single bank does not have a significant effect on the equilibrium in this market then the pricing decisions of some banking products may be almost not related. In other words, banks maximize their profits in deposit and loan markets separately by considering the interest rate in the interbank lending market,[5] instead of solving the total profit maximization problem. If it is so then the unobservable characteristics of saving contracts have no effect on the interest rates of loans, mortgages, etc. However, the interest rates in lending and saving deposit markets are related to the interbank market equilibrium interest rate. Hence, the interest rates on mortgages and consumer loans of the same bank may be used as instruments in the model.

Secondly, the approach of Nevo (2001) may be followed, i.e. the saving deposit interest rates from other markets (countries) can be used as instruments too. A bank customer usually signs a saving deposit contract with the banks that perform their activities in her country of residence. Therefore, it is very unlikely that banks compete for the saving deposits internationally. However, if there is a joint interbank lending market between several countries, e.g. Eurozone, then the saving deposit interest rates in the analysed country may be instrumented by the aggregated interest rates on saving deposits, mortgages, etc. from other countries.

5.3 Model specification

5.3.1 Customer behaviour

In this section we assume that all bank customers search for the contracts that suit them the best, and a bank customer may search any number of credit institutions. We put some restrictions on the number of searches in section 5.4. This will be done in order to make the simulation and estimation process less time consuming.

[5] EURIBOR, LIBOR, etc.

However, in this section we derive the general market share expressions that later are specialized for the restricted search case.

Consider a bank customer i, who considers signing a saving deposit contract. The customer can sign a saving contract with one of J banks. Every bank offers several different saving contracts. We denote the set of the saving contracts that are offered by bank j in period t as $\mathcal{Z}_{jt}$. If customer i signs contract $k \in \mathcal{Z}_{jt}$ with bank j in period t then she gets the following utility:

$$u_{ki} = \mathbf{x}_k \boldsymbol{\beta} + \xi_k + \varepsilon_{ki} = \delta_k + \varepsilon_{ki} \tag{5.1}$$

where $\mathbf{x}_k$ is the vector of observable characteristics of alternative k at time t, $\boldsymbol{\beta}$ is a parameter vector, ξ_k shows the contract characteristics that are not observable for the econometrician but known by the customer, and ε_{ki} is a customer and contract specific error term, which is distributed according to the extreme value type I distribution.

Customer i may choose an outside option, which consists of not signing any saving contract and keeping her money in her current account or in terms of cash at home. In this case her utility equals

$$u_{0i} = \delta_{0i} + \varepsilon_{0i}$$

where δ_0 is normalized to 0.

If a customer knows all the utilities that are available in the market then she chooses the one with the highest utility. A typical bank customer knows the deterministic part δ of utility of every saving contract before she visits a credit institution. However, the customer needs to learn customer and contract specific terms ε. Information about ε is costly. Thus, the customer has to decide what alternatives she wants to learn fully about.

A typical bank customer compares the gains from an additional search and the search costs when she makes a decision whether to terminate her search or continue searching further. Let us assume that the customer is in bank j where she has observed an alternative that gives her utility $\tilde{u}_{ji}$, which is the highest observed utility so far. The customer considers searching in bank $h \neq j$, and $\tilde{u}_{hi}$ refers to the maximum utility from bank h as , i.e. $\tilde{u}_{hi} = \max\{u_{li}\}_{l \in \mathcal{Z}_{ht}}$. Then the customer is indifferent between searching in bank h or terminating her search in bank j if the following equality is satisfied.[6]

[6] See Weitzman (1979) for the derivation of the optimal stopping rule.

$$c + \nu_{hi} = \int_{\tilde{u}_{ji}}^{\infty} \left(\tilde{u}_{hi} - \tilde{u}_{ji}\right) dG_{hi}\left(\tilde{u}_{hi}\right) \tag{5.2}$$

where $c + \nu_{hi}$ is a customer specific search cost for bank h in period t and $G_{hi}\left(\tilde{u}_{hi}\right)$ is the distribution function of $\tilde{u}_{hi}$:

$$\begin{aligned} G_{hi}\left(\tilde{u}_{hi}\right) &= \int_{-\infty}^{\tilde{u}_{hi}} ... \int_{-\infty}^{\tilde{u}_{hi}} \underbrace{dF_u\left(u_{wi}\right) ... dF_u\left(u_{1i}\right)}_{\mathcal{Z}_{ht}} \\ &= \int_{-\infty}^{\tilde{u}_{hi}-\delta_1} ... \int_{-\infty}^{\tilde{u}_{hi}-\delta_w} dF\left(\varepsilon_{wi}\right) ... dF\left(\varepsilon_{1i}\right) \\ &= \prod_{l \in \mathcal{Z}_{ht}} F\left(\tilde{u}_{hi} - \delta_l\right) \end{aligned}$$

Since ε is distributed according to the type I extreme value distribution, we have

$$\begin{aligned} G_{hi}\left(\tilde{u}_{hi}\right) = \exp\left\{ -\sum_{l \in \mathcal{Z}_{ht}} e^{\delta_l - \tilde{u}_{hi}} \right\} = \\ = \exp\left\{ -e^{-\tilde{u}_{hi}} \sum_{l \in \mathcal{Z}_{ht}} e^{\delta_l} \right\} = \\ = \exp\left\{ -e^{-\tilde{u}_{hi}} \gamma_h \right\} \end{aligned}$$

where $\gamma_h = \sum_{l \in \mathcal{Z}_{ht}} e^{\delta_l}$.

We introduce a scalar $\bar{u}_{hi}$ such that if $\tilde{u}_{ji} = \bar{u}_{hi}$ then (5.2) is satisfied.[7] The value of $\bar{u}_h$ is independent from the characteristics of bank j and depends on the characteristics of bank h only. Therefore, this value is called the *reservation utility* at bank h. If $\tilde{u}_j > \bar{u}_h$ then the gain from searching in bank h is less than the search cost[8] and a customer prefers to terminate her search in bank j . On the contrary, if $\tilde{u}_j < \bar{u}_h$ then a customer wants to pay $c + \nu_h$ to learn about the offers of bank h.

The reservation utility for every bank in the market can be computed from equation (5.2). According to Weitzman (1979), a rational sequentially searching bank customer should rank all banks according to their reservation utilities from the highest to the lowest and start searching from the top of the list. Thus, if a bank customer is in bank j and later searches in bank h then $\bar{u}_h > \max\left\{\bar{u}_l\right\}_{l=1,..J \setminus \{h,j\}}$.

[7] Consumer specific index i at c_{hi} and $\bar{u}_{hi}$ will be omitted from here on.
[8] The RHS of (5.2) is less than the LHS.

5.3.2 Market share derivation

Bank j signs contract $k \in \mathcal{Z}_{jt}$ with two types of customers. Firstly, the credit institution signs contract k with the customers who have their current accounts in bank j. These customers are referred as "*the own customers*" of the bank from here on. The second type of the customers who sign contract k with bank j have their accounts in the banks other than bank j. We refer to these customers as "*the alien customer*s" of bank j. We introduce variable λ_j to label the share of the customers who have their current accounts in bank j, and split the derivation of the market share of contract k in two parts according to the customer type. Later these parts are aggregated for the final expression of the market share.

An own customer signs contract k with bank j in several cases. Firstly, she may choose contract k after she observes the offers of bank j and learns the utility of an outside option without searching other credit institutions. Secondly, the customer may continue searching beyond bank j, terminate her search after observing the offers of several (not all) banks and decide to sign contract k with bank j. Lastly, the customer may search all the banks in the market and decide that contract k with bank j is the best option for her.

Let us assume that the search order is predetermined, i.e. all customers follow the same search order. The customers start from the banks where they have their current accounts. Then they continue searching the banks in a numerically ascending order. More particularly, if a bank customer starts searching from bank j then she may visit bank 1 after bank j. If the customer does not find a satisfactory offer in both bank j and bank 1 then she searches bank 2, and so on till bank $j-1$. If the customer searches beyond bank $j-1$ then she goes to bank $j+1$ and may continue with bank $j+2$, bank $j+3$, etc. up to bank J.

An own customer accepts contract k in bank j without searching bank 1 if the utility of contract k is the highest among all observed utilities and is above $\bar{u}_1$. We label this probability as P_k^{o1} where the superscript o indicates an own customer.

$$
\begin{aligned}
P_k^{o1} &= \Pr\left[u_k = \max\left\{u_l\right\}_{l \in \mathcal{Z}_{jt} \cup \{0\}} \text{ and } u_k > \bar{u}_1\right] \\
&= \int_{\bar{u}_1}^{\infty} \prod_{l \in \mathcal{Z}_{jt} \setminus k \cup \{0\}} F_u\left(u_k\right) dF_u\left(u_k\right) \\
&= \int_{\bar{u}_1 - \delta_k}^{\infty} \prod_{l \in \mathcal{Z}_{jt} \setminus k \cup \{0\}} F\left(u_k - \delta_l\right) dF\left(\varepsilon\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \int_{\bar{u}_1-\delta_k}^{\infty} \exp\left\{-e^{-u_k} \sum_{l\in\mathcal{Z}_{jt}\backslash k\cup\{0\}} e^{\delta_l}\right\} \exp\left\{-e^{-\varepsilon}\right\} e^{-\varepsilon} d\varepsilon \\
&= \int_{\bar{u}_1-\delta_k}^{\infty} \exp\left\{-e^{-\varepsilon}\left(\sum_{l\in\mathcal{Z}_{jt}\backslash k\cup\{0\}} e^{\delta_l-\delta_k}+1\right)\right\} e^{-\varepsilon} d\varepsilon \\
&= \frac{1}{e^{-\delta_k}\left(\gamma_j+1\right)} \int_{\bar{u}_1-\delta_k}^{\infty} \exp\left\{-e^{-\varepsilon+ln\left(e^{-\delta_k}\left(\gamma_j+1\right)\right)}\right\} e^{-\varepsilon+ln\left(e^{-\delta_k}\left(\gamma_j+1\right)\right)} d\varepsilon \\
&= \frac{1}{e^{-\delta_k}\left(\gamma_j+1\right)} \int_{\bar{u}_1-\delta_k-ln\left(e^{-\delta_k}\left(\gamma_j+1\right)\right)}^{\infty} \exp\left\{-e^{-\varepsilon}\right\} e^{-\varepsilon} d\varepsilon \\
&= \frac{e^{\delta_k}}{\left(\gamma_j+1\right)}\left(1-F\left(\bar{u}_1-\ln\left(\gamma_j+1\right)\right)\right)
\end{aligned}
$$

The customer who is not satisfied with the offer of bank j continues searching bank 1. Her decision whether to search bank 2 or to choose the best observed offer right away depends on the value of the best observed offer and the reservation utility $\bar{u}_2$. The reservation utility at bank 2 is less than the reservation utility at bank 1, i.e. $\bar{u}_1 > \bar{u}_2$. The customer has searched bank 1 because $\tilde{u}_j \leq \bar{u}_1$. Thus, the customer may terminate her search after seeing the offers of bank 1 and choose contract $k \in \mathcal{Z}_{jt}$. This happens if the utility of alternative k is less than $\bar{u}_1$ but it is above $\bar{u}_2$ and it is the highest utility among the observed utilities. We label this probability as P_k^{o2}.

$$
\begin{aligned}
P_k^{o2} &= \Pr\left[u_k = \max\left\{u_l\right\}_{l\in\mathcal{Z}_{jt}\cup\mathcal{Z}_{1t}\cup\{0\}} \text{ and } \bar{u}_1 > u_k > \bar{u}_2\right] \\
&= \int_{\bar{u}_2}^{\bar{u}_1} \prod_{l\in\mathcal{Z}_{jt}\backslash k\cup\mathcal{Z}_{1t}\cup\{0\}} F_u\left(u_k\right) dF_u\left(u_k\right) \\
&= \int_{\bar{u}_2-\delta_k}^{\bar{u}_1-\delta_k} \prod_{l\in\mathcal{Z}_{jt}\backslash k\cup\mathcal{Z}_{1t}\cup\{0\}} F\left(u_k-\delta_l\right) dF_\varepsilon\left(\varepsilon\right) \\
&= \int_{\bar{u}_2-\delta_k}^{\bar{u}_1-\delta_k} \exp\left\{-e^{-u_k} \sum_{l\in\mathcal{Z}_{jt}\backslash k\cup\{0\}\cup\mathcal{Z}_{1t}} e^{\delta_l}\right\} \exp\left\{-e^{-\varepsilon}\right\} e^{-\varepsilon} d\varepsilon \\
&= \frac{e^{\delta_k}}{\left(\gamma_j+\gamma_1+1\right)}\left(F\left(\bar{u}_1-\ln\left(\gamma_j+\gamma_1+1\right)\right)-F\left(\bar{u}_2-\ln\left(\gamma_j+\gamma_1+1\right)\right)\right)
\end{aligned}
$$

The customer may search bank 2 and then return to bank j for contract k without searching further. This happens if u_k is the highest utility among the observed utilities, it is less than $\bar{u}_2$ but it is higher than the reservation utility of the next searched bank $(\bar{u}_3)$. The expression of this probability is very similar to the expression of

P_k^{o2}. The only difference is that the values of $\bar{u}_1$ and $\bar{u}_2$ must be replaced with the values of other reservation utilities ($\bar{u}_2$ and $\bar{u}_3$), and γ_2 has to be added in the expressions under the logarithms and in the denominator of the fraction in the front. The customer may return to bank j and sign contract k after every searched bank. Thus, we have a sequence of probabilities that are similar to P_k^{o2}.

The own customer may search all the banks in the market and still return to bank j and sign contract k. The last visited bank is bank J. Then the customer signs contract k if u_k is less than $\bar{u}_J$ and is the highest available utility in the market. We name the set of alternatives $\mathcal{Z}_{jt} \cup \mathcal{Z}_{1t} \cup \mathcal{Z}_{2t} \cup ... \cup \mathcal{Z}_{Jt} \cup \{0\}$ as $\mathcal{Z}^{jt}$. Then the probability that an own customer takes contract k after searching all the banks can be written as follows

$$\begin{aligned} P_k^{oJ} &= \Pr\left[u_k = \max\left\{u_l\right\}_{l \in \mathcal{Z}^{jt}} \text{ and } u_k < \bar{u}_J\right] \\ &= \frac{e^{\delta_k}}{\sum_{l=1}^{J} \gamma_l + 1} F\left(\bar{u}_J - \ln\left(\sum_{l=1}^{J} \gamma_l + 1\right)\right) \end{aligned}$$

Thus, conditional on that the order according to which a bank customer visits the other $J-1$ banks is known, the probability that an own customer signs contract k can be expressed as follows

$$P_k^o = \sum_{l=1}^{J} P_k^{ol} \tag{5.3}$$

The second part of contract k demand consists of the alien customers who arrive at bank j from other banks and sign contract k. Let us assume that an alien customer starts searching from bank 1. Then bank j is visited in the j^{th} position, and the next bank that the customer would visit after bank j is bank $j+1$. The customers have left banks up to bank $j-1$ without signing any contract because the maximum utilities in these banks have been less than the reservation utility at bank j, i.e. $\max\left\{u_l\right\}_{l=\mathcal{Z}_{1t}\cup...\cup\mathcal{Z}_{(j-1)t}\cup\{0\}} < \bar{u}_j$. These customers may sign contract k if they terminate their search in bank j or continue searching further and return to bank j for contract k.

If the alien customers terminate their search in bank then j they sign contract k in two cases. First of all, these customers sign the contract if u_k is the highest utility in bank j and is higher than $\bar{u}_j$. Secondly, the alien customers terminate their search in bank j and sign contract k if $\bar{u}_{j+1} < u_k < \bar{u}_j$ and u_k is the highest observed utility. We label the probability that an alien customer from bank 1 has arrived at bank j, terminated her search here and signed contract k as $P_{k(1)}^{aj}$. The index next to k shows the bank where the customer started searching.

$$
\begin{aligned}
P_{k(1)}^{aj} &= \Pr\left[u_k \geq \max\left\{u_l, \bar{u}_j\right\}_{l\in\mathcal{Z}_{jt}} \text{ and} \max\left\{u_l\right\}_{l\in\mathcal{Z}_{1t}\cup\ldots\cup\mathcal{Z}_{(j-1)t}\cup\{0\}} < \bar{u}_j\right] \\
&+\Pr\left[\bar{u}_{j+1} < u_k < \bar{u}_j \text{ and} \max\left\{u_l\right\}_{l\in\mathcal{Z}_{1t}\cup\ldots\cup\mathcal{Z}_{jt}\cup\{0\}} \leq u_k\right] \\
&= \prod_{q=1}^{j-1} G_q\left(\bar{u}_j\right) F\left(\bar{u}_j\right) \frac{e^{\delta_k}}{\gamma_j}\left(1 - F\left(\bar{u}_j - \ln\left(\gamma_j\right)\right)\right) \\
&+ \frac{e^{\delta_k}}{\sum_{q=1}^{j}\gamma_q + 1}\left(F\left(\bar{u}_j - \ln\left(\sum_{q=1}^{j}\gamma_q + 1\right)\right) - F\left(\bar{u}_{j+1} - \ln\left(\sum_{q=1}^{j}\gamma_q + 1\right)\right)\right)
\end{aligned}
$$

An alien customer from bank 1 continues searching beyond bank j if all observed utilities are less than $\bar{u}_{j+1}$. The customer may search bank $j+1$ and return back to bank j for contract k without seeing the offers of bank $j+2$. This happens if $\bar{u}_{j+1} > u_k > \bar{u}_{j+2}$ and u_k is the highest observed utility. We name this probability as $P_{k(1)}^{aj+1}$

$$
\begin{aligned}
P_{k(1)}^{aj+1} &= \Pr\left[\bar{u}_{j+2} < u_k < \bar{u}_{j+1} \text{ and} \max\left\{u_l\right\}_{l\in\mathcal{Z}_{1t}\cup\ldots\cup\mathcal{Z}_{(j+1)t}\cup\{0\}} \leq u_k\right] \\
&= \frac{e^{\delta_k}}{\sum_{q=1}^{j+1}\gamma_q + 1}\left(F\left(\bar{u}_{j+1} - \ln\left(\sum_{q=1}^{j+1}\gamma_q + 1\right)\right)\right. \\
&\left. - F\left(\bar{u}_{j+2} - \ln\left(\sum_{q=1}^{j+1}\gamma_q + 1\right)\right)\right)
\end{aligned}
$$

The reservation utilities of the banks are ranked in a decreasing order. Thus, an alien customer from bank 1 may return back to bank j and sign contract k from any credit institution that is searched beyond bank j. The probability of this event equals the probability that u_k is less than the reservation utility at the last searched bank, it is higher than the reservation utility at the subsequent bank, and it is the highest utility among all observed utilities. Consequently, the expression of the probability that an alien customer from bank 1 returns to bank j for contract k from any bank that is searched beyond bank j is similar to the expression $P_{k(1)}^{aj+1}$. However, the values of the reservation utilities and the sum of γs have to be adjusted accordingly. Finally, an alien customer may sign contract k after she observes all the contracts in the market. The probability of this event is identical to the probability P_k^{oJ}. We refer to the probability that an alien customer from bank 1 returns to bank j for contract k after visiting all the banks in the market as $P_{k(1)}^{aJ}$. The total probability that an alien customer from bank 1 signs contract k with bank j is the sum of $J-j+1$ probabilities. We label this sum as $P^{a}_{k(1)}$

$$P^{a}_{k(1)} = \sum_{q=j}^{J} P^{aq}_{k(1)}$$

Bank j may sign contract k with the alien customers who start searching from any of $J-1$ banks. Therefore, the probability that bank j signs contract k with an alien customer, given the search order, equals the sum of $J-1$ probabilities similar to $P^{a}_{k(1)}$ weighted by the appropriate values of λ. We name the probability as P^{a}_{k}.

$$P^{a}_{k} = \sum_{z=1}^{J-1} \lambda_z P^{a}_{k(z)}$$

The order of visited banks affects the expressions of the probability that own and alien customers sign saving contract k. This happens because the combinations of γs and the values of the reservation utilities in the probabilities P^{ol}_{k} and $P^{a}_{k(l)}$, $l \in [1;\, J]$ depend on the search order. It has been stated that the search cost varies across the banks and their customers. The random parameter in the search cost expression implies that there is a positive probability of every search order in the market, i.e. there is a chance that a customer follows any of $(J-1)!$ different search orders.

Let us consider the own customers of bank j who signs contract k with bank j. The chance that these customers follow the search order $j, 1, 2, ...j-1, j+1, j+2, ...J$ equals

$$\Pr\left[\bar{u}_1 > \bar{u}_2 > ... > \bar{u}_{j-1} > \bar{u}_{j+1} > ... > \bar{u}_J\right] \tag{5.4}$$

Then the probability that the own customers of bank j sign contract k and follow the specified order equals the following $J-1$ dimensional integral

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\bar{u}_1}\int_{-\infty}^{\bar{u}_2} ... \int_{-\infty}^{\bar{u}_{J-1}} P^{o}_{k} dF_{\bar{u}}(\bar{u}_J)...dF_{\bar{u}}\left(\bar{u}_3\right) dF_{\bar{u}}(\bar{u}_2) dF_{\bar{u}}(\bar{u}_1)$$

The own customers may follow any possible order. The set of indexes $\{1,\, 2,\, ...,\, J-1\}$ may be arranged in $(J-1)!$ orders. We denote the set of orders by Γ. Then, $\Gamma\,(q,\, i)$ shows the i^{th} element of the q^{th} order. Then the probability, that an own customer of bank j signs contract k is

$$p_k^o = \sum_{q=1}^{(J-1)!} \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}_{\Gamma(q,1)}} \int_{-\infty}^{\bar{u}_{\Gamma(q,2)}} ... \int_{-\infty}^{\bar{u}_{\Gamma(q,J-2)}} P_k^o dF_{\bar{u}}(\bar{u}_{\Gamma(q,J-1)})...dF_{\bar{u}}(\bar{u}_{\Gamma(q,2)})dF_{\bar{u}}(\bar{u}_{\Gamma(q,1)})$$

Bank j may be visited in any position from 2 to J by the alien customers from bank 1 with some positive probability. This, implies that the probability that the alien customers from bank 1 sign contract k with bank j is the sum of $(J-2)!(J-1)$ multidimensional integrals. In order to illustrate this statement let us assume that there are four banks in the market and $j = 2$. Then the alien customers who start searching from bank 3 and sign contract k with bank 2 may follow this list of search orders

$$3 \to \mathbf{2} \to 1 \to 4$$
$$3 \to \mathbf{2} \to 4 \to 1$$
$$3 \to 1 \to \mathbf{2} \to 4$$
$$3 \to 4 \to \mathbf{2} \to 1$$
$$3 \to 4 \to 1 \to \mathbf{2}$$
$$3 \to 1 \to 4 \to \mathbf{2}$$

Then the probability that an alien customer who starts searching from bank 3 and signs contract k with bank 2 equals the following expression

$$\begin{aligned} p_{k(3)}^a = & \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}_2} \int_{-\infty}^{\bar{u}_1} P_{k(3)}^a dF_{\bar{u}}\left(\bar{u}_4\right) dF_{\bar{u}}\left(\bar{u}_1\right) dF_{\bar{u}}\left(\bar{u}_2\right) \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}_2} \int_{-\infty}^{\bar{u}_4} P_{k(3)}^a dF_{\bar{u}}\left(\bar{u}_1\right) dF_{\bar{u}}\left(\bar{u}_4\right) dF_{\bar{u}}\left(\bar{u}_2\right) \\ & + ... \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}_1} \int_{-\infty}^{\bar{u}_4} P_{k(3)}^a dF_{\bar{u}}\left(\bar{u}_2\right) dF_{\bar{u}}\left(\bar{u}_4\right) dF_{\bar{u}}\left(\bar{u}_1\right) \end{aligned}$$

We introduce an additional index z in the notation of the elements of Γ for the general expression of $p_{k(1)}^a$. The triple (q, z, i) denotes the i^{th} element of the q^{th} arrangement of $J-1$ indexes, given that bank j is sampled on the z^{th} position. Then the probability that bank j signs contract k with the alien customers from bank 1 is

$p^a_{k(1)}$:

$$p^a_{k(1)} = \sum_{z=1}^{J-1} \sum_{q=1}^{(J-2)!} \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}_{\Gamma(q,z,1)}} ... \int_{-\infty}^{\bar{u}_{\Gamma(q,z\,J-2)}} P^a_{k(1)} dF_{\bar{u}} \left(\bar{u}_{\Gamma(q,z,J-1)} \right) ... dF_{\bar{u}} \left(\bar{u}_{\Gamma(q,z,1)} \right)$$

The multidimensional integrals make the estimation of the model quite complicated. The analytical solutions of the integrals are not possible because it is not possible to get the analytical expression of $\bar{u}_h$ from (5.2). However, the model is still tractable if there are a few banks in the market. Unfortunately some restrictions on the search order or the number of searches are necessary if the number J is relatively large.

Not the quantity of contracts but the sum of money matters in a banking sector. We follow the similar methodology to the one of Ishii (2005) and Zhou (2008) in defining the market shares of the saving deposit contracts. Therefore, we define the market share of contract k as the ratio between the expected deposits of contract k and the total expected sum of deposits in the market. Then the share of saving deposits of contract k may be written as follows

$$s_k = \frac{\int_y y \left(\lambda_j p^o_k + \sum_{q=1}^{J-1} \lambda_q p^a_{k(q)} \right) dF_y(y)}{\int_y y dF_y(y)} = \lambda_j p^o_k + \sum_{q=1}^{J-1} \lambda_q p^a_{k(q)}$$

where y denotes the deposit size.

5.4 Model simulation

We simulate the saving deposit demand model and estimate its parameters in this section. It is very computationally intensive to perform numerical integration of the market shares over the distribution of the search costs because it involves solving many non-linear search rule equations. Therefore, we put some restrictions on the search behaviour of consumers, which simplify the multidimensional integration problem and let to speed up the estimation process. We assume that there are three types of bank customers in the simulated market. The first group of bank customers have zero search costs and compare all saving deposit contracts before they sign any of them. The second group of the customers do not search beyond the banks where they have their current accounts and the third group of bank customers observe the offers of two banks at most.

The three types of banks customers are distinguished because of the particularities of the Dutch retail banking market. According to OECD (2007) report, all Dutch households kept their current accounts in one of four biggest banks in 2007.[9] In addition, these banks had 90% of all saving accounts. It is very likely that a typical bank customer searches for a saving contract only among four biggest banks. However, the fact that about one tenth of all saving accounts are distributed among other smaller banks suggests that there is a fraction of actively searching bank customers in the market. Moreover, the survey data showed that about 21% of Dutch bank customers would have switched from one bank to another to get all banking services from only one credit institution in 2006.[10] Consequently, we assume that there are bank customers who have zero search costs, the customers who do not search beyond the bank where they have their current accounts and the customers who perform a limited number of searches.[11]

The first group comprises of 10% of all the customers. These customers compare all the saving contracts in the market and choose the ones with the highest utilities. Hence, the probability that they sign contract k is as follows

$$P_k^0 = \frac{e^{\delta_k}}{\sum_{l=1}^{J} \gamma_l + 1}$$

20% of all the bank customers do not search beyond the bank where they have their current accounts. These customers choose between the contracts of the banks where they have their current accounts and an outside option. Therefore, the probability that a non-searching customer signs contract k is

$$P_k^n = \frac{e^{\delta_k}}{\gamma_j + 1}$$

This customer must have a current account in bank j

Finally, the remaining customers may search beyond the banks where they have their current accounts. However, they do not search more than one more bank. In other words, these customers observe the offers of two banks at most. The introduction of zero search cost and not-searching bank customers brings in some confusion to the classification of the bank customers that has been used in section 5.3.2. This is because the customers who have their current accounts in bank j can belong to any

[9] They are ABN AMRO/Fortis, Rabobank, ING, SNS Bank.

[10] Lelieveldt (2006)

[11] Moraga-González and Wildenbeest (2008) estimated the shares of consumers who searched for one, two, etc. up to all prices of memory chips in their non-sequential search model. They found that the estimated shares of the consumers who observed only one, two, three and all the prices were significantly above zero.

of three customer groups that have been introduces in this section. In order to draw the parallel between the derived probabilities in section 5.3.2 and the corresponding probabilities in this section, we make the distinction between *"own customers"* and *"alien customers"* just for the customers who have positive search costs and may search beyond the first bank. In other words, zero search cost customers and not-searching customers are not divided into any subgroups.

The probability that the own customers of bank j sign contract k, given that bank q is to be searched next, equals

$$
\begin{aligned}
P_k^o = & \frac{e^{\delta_k}}{(\gamma_j + 1)} \left(1 - F\left(\bar{u}_q - \ln\left(\gamma_j + 1\right)\right)\right) \\
& + \frac{e^{\delta_k}}{\gamma_j + \gamma_q + 1} F\left(\bar{u}_q - \ln\left(\gamma_j + \gamma_q + 1\right)\right)
\end{aligned}
$$

The own customer of bank j may search any of $J-1$ banks. Therefore, the probability than an own customer of bank j signs contract k is as follows

$$
p_k^o = \sum_{q=1}^{J-1} \int_{-\infty}^{\infty} \left(\prod_{l=1, l \notin \{q,j\}}^{J-2} F_{\bar{u}_l}\left(\bar{u}_q\right) \right) P_k^o\left(\bar{u}_q, \gamma_q, \gamma_j, \delta_k\right) dF_{\bar{u}_q}\left(\bar{u}_q\right)
$$

If bank j does not have its own customers then $p_k^o = 0$.

Now let us consider the alien customers who arrive at bank j from bank $q \neq j$. The probability that these customers sign contract k is

$$
\begin{aligned}
P_{k(q)}^a = & G_q\left(\bar{u}_j\right) F\left(\bar{u}_j\right) \frac{e^{\delta_k}}{\gamma_j} \left(1 - F\left(\bar{u}_j - \ln\left(\gamma_j\right)\right)\right) \\
& + \frac{e^{\delta_k}}{\gamma_q + \gamma_j + 1} F\left(\bar{u}_j - \ln\left(\gamma_q + \gamma_j + 1\right)\right)
\end{aligned}
$$

Alien customers can arrive from $J-1$ banks. They search bank j if $\bar{u}_j > \max\left\{\bar{u}_l\right\}_{l \neq \{j,q\}}$. Hence, the probability that alien customers arrive at bank j and sign contract k is

Then

$$
p_k^a = \sum_{q=1}^{J-1} \int_{-\infty}^{\infty} \left(\prod_{l=1, l \notin \{q,j\}}^{J-2} F_{\bar{u}_l}\left(\bar{u}_j\right) \right) \lambda_q P_{k(q)}^a\left(\bar{u}_j, \gamma_q, \gamma_j, \delta_k\right) dF_{\bar{u}_j}\left(\bar{u}_j\right)
$$

The market share of contract k is the sum of the probabilities that the customers from all three above mentioned groups sign contract k:

$$s_k = \frac{1}{10} P_k^0 + \frac{1}{5} \lambda_j P_k^n + \frac{7}{10} \left(\lambda_j p_k^o + p_k^a \right)$$

During the model simulation we assumed that there are 10 banks in the market. Five banks had their own customers, and five banks signed saving contracts with alien customers only. The vector of current account market shares $(\boldsymbol{\lambda})$ for five banks was generated as follows. Five random draws were taken from $U(0, 1)$ and $\lambda_i = draw_i / \sum_j draw_j$.

Matrix $\boldsymbol{X}$ had five columns in the simulated model. We included the yearly interest rates, a saving account dummy, the share of banks' assets, and two dummies that showed whether a bank had own customers or not. There were twelve periods in our model and every bank offered three types of deposits (a saving account, a half-year fixed term contract and an one-year fixed contract). Hence, there were $10 \times 12 \times 3 = 360$ observations.

The share of assets per period was generated similarly as $\boldsymbol{\lambda}$, i.e. by drawing from $U(0, 1)$ and normalizing by the sum of the drawn numbers. The asset variable varied every three periods. In other words, one generated share of assets for bank j was the same in the periods from 1 to 3, there was the same share of assets in the periods from 4 to 6, etc. The repetitive values of the asset shares were used because banks report the newly accepted deposits and their interest rates monthly, however, the financial accounting data frequency is one quarter. The asset share variable was generated once and remained fixed for all simulations.

The interest rates of the contracts were affected by the interbank market lending interest rate $(\bar{r})$ and ξ. The interbank market interest rate vector was generated as a random draw from $U(0, 1)$ for each period (twelve draws in total). The vector $\bar{\boldsymbol{r}}$ was generated once and was fixed for all model simulations. The unobserved contract characteristics were drawn independently from $\mathcal{N}(0, 1)$ for each observation. A new set of $\boldsymbol{\xi}$ was drawn for every simulation. Hence, the interest rates on saving contracts varied with the simulations too.

The interest rates on saving contracts were simulated as follows. We label the vector of the interest rates for saving accounts $\boldsymbol{r_s}$, the vector of the interest rates for half-year fixed term deposits $\boldsymbol{r_{0.5}}$ and the vector of the interest rate for one-year fixed term deposits $\boldsymbol{r_1}$. Then the generation of the interest rate vectors for period t was performed by using these expressions

$$\begin{aligned}
\boldsymbol{r_{st}} &= \frac{1}{100}\exp\left\{\boldsymbol{\xi_{st}}\right\} + \frac{1}{2}\bar{r}_t \otimes \boldsymbol{i_{1\times 10}} \\
\boldsymbol{r_{0.5t}} &= \frac{1}{100}\exp\left\{\boldsymbol{\xi_{0.5t}}\right\} + 2\bar{r}_t \otimes \boldsymbol{i_{1\times 10}} \\
\boldsymbol{r_{1t}} &= \frac{1}{100}\exp\left\{\boldsymbol{\xi_{1t}}\right\} + 4\bar{r}_t \otimes \boldsymbol{i_{1\times 10}}
\end{aligned}$$

where $\boldsymbol{i_{1\times 10}}$ is a 1×10 vector of ones.

We follow the approach of Nevo (2001) for the model instruments. In other words we assume that it is possible to use the interest rates on the deposits (loans) from other markets (countries). These interest rates are not affected by the characteristics of the contracts and banks in the analyzed market, however, they are affected by the interbank lending market equilibrium interest rate $\bar{r}$. Hence, we generated one instrument z for the model which was simulated as the function of $\bar{r}$ and some random variables. We denote the vector of instruments for period t as $\boldsymbol{z_t}$. Then

$$\boldsymbol{z_t} = \left(2\bar{r}_t \otimes \boldsymbol{i_{1\times 30}}\right) \circ \boldsymbol{A} + 0.01\boldsymbol{B}$$

where $\boldsymbol{A}$ is a 1×30 vector which element $a_i = 1/l_i$ and $l_i \sim U(1, 2)$, $\boldsymbol{B}$ is 1×30 vector which element $b_i \sim \mathcal{N}(0, 1)$.

The model estimation was run according to BLP procedure for random coefficient logit models, i.e. we minimized the GMM criterion function $Q(c, \boldsymbol{\beta})$ with respect to c and $\boldsymbol{\beta}$:

$$\left(\hat{\boldsymbol{\beta}}, \hat{c}\right) = \arg\min_{c,\boldsymbol{\beta}} Q(c, \boldsymbol{\beta}) = \boldsymbol{\omega}(c, \boldsymbol{\beta})^T \boldsymbol{Z}\left(\boldsymbol{Z}^T \boldsymbol{\Omega} \boldsymbol{Z}\right)^{-1} \boldsymbol{Z}^T \boldsymbol{\omega}(c, \boldsymbol{\beta})$$

where $\boldsymbol{Z}$ is an instrument matrix, $\boldsymbol{\Omega}$ stands for the variance-covariance matrix and $\boldsymbol{\omega}(c, \boldsymbol{\beta})$ is

$$\boldsymbol{\omega}(c, \boldsymbol{\beta}) = \boldsymbol{\delta} - \boldsymbol{X}\boldsymbol{\beta}$$

The parameter vector $\boldsymbol{\beta}$ was integrated out in the estimation procedure by

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{X}^T \boldsymbol{Z} \boldsymbol{\Omega}^{-1} \boldsymbol{Z}^T \boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \boldsymbol{Z} \boldsymbol{\Omega}^{-1} \boldsymbol{Z}^T \boldsymbol{\delta}(\hat{c})$$

and $\boldsymbol{\delta}(\hat{c})$ was obtained by solving the system of equations $\boldsymbol{s} - \boldsymbol{s}(\boldsymbol{\delta}, \hat{c}) = 0$ Hence, the search for the minimum of the GMM objective function was performed with respect to c.[12]

The estimation consisted of three optimization loops. The inner loop consisted of solving the search rule equations (5.2) for $\bar{\boldsymbol{u}}$, given the values of γ and $c+\boldsymbol{\nu}$. The middle loop involved solving the system of equations $\boldsymbol{s}-\boldsymbol{s}(\delta,\hat{c})=0$ to get $\delta(c)$. The final loop was the minimization of the objective function.

We could not solve (5.2) for the value of $\bar{u}_h$ explicitly. Hence, we could not write the expressions for the distribution functions $F_{\bar{u}}(\bar{u})$. Therefore, the calculation of the market shares went as follows. We drew 3000 values of ν independently from the uniform distribution $U(-1,1)$. These values were grouped in 300 ten-dimensional vectors. Element j, $j\in[1,10]$ of vector $\boldsymbol{\nu}_i$ was used to calculate $\bar{u}_{ji}$. Thus, for each $\boldsymbol{\nu}_i$, $i\in[1,300]$ we had a vector $\bar{\boldsymbol{u}}_i$.

When the values of $\bar{\boldsymbol{u}}_i$ were known, then the ranking of the reservation utilities was known and we could write down the expressions of the market shares. This procedure was repeated for 300 times. Therefore, we got 300 expressions of the market share of each contract. All 300 market shares of one contract were added up and divided by 300 to obtain the final expression of the contract's market share. In other words, the expressions of p_k^o and p_k^a were calculated according to these formulas

$$
\begin{aligned}
p_k^o &= \frac{1}{300}\sum_{i=1}^{300}\sum_{q=1}^{9}\mathbf{1}_q\left(\bar{u}_{qi}\right)P_k^o\left(\bar{u}_{qi},\gamma_q,\gamma_j,\delta_k\right)\\
p_k^a &= \frac{1}{300}\sum_{i=1}^{300}\sum_{q=1}^{9}\mathbf{1}_j\left(\bar{u}_{ji}\right)\lambda_q P_{k(q)}^a\left(\bar{u}_{ji},\gamma_q,\gamma_j,\delta_k\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{1}_q\left(\bar{u}_{qi}\right) &= \begin{cases}1, & \bar{u}_{qi}=\max\left\{\bar{u}_{li}\right\}_{l\neq j}\\ 0, & \bar{u}_{qi}\neq\max\left\{\bar{u}_{li}\right\}_{l\neq j}\end{cases}\\
\mathbf{1}_j\left(\bar{u}_{ji}\right) &= \begin{cases}1, & \bar{u}_{ji}=\max\left\{\bar{u}_{li}\right\}_{l\neq q}\\ 0, & \bar{u}_{ji}\neq\max\left\{\bar{u}_{li}\right\}_{l\neq q}\end{cases}
\end{aligned}
$$

We estimated the model by using *R* environment and explored several optimization routines for the minimization of the GMM objective function and the inversion of the market shares. More particularly, we used these *R* packages: *BB*

[12] See Nevo (2000) for more details.

Table 5.1. Simulation results

Variable name	True value	Case I[a]		Case II[b]		Case III[b]	
		Mean	StD	Mean	StD	Mean	StD
Dummy 1[c]	-1.2	-1.094	1.154	-2.395	0.797	-1.521	1.179
Dummy 2[d]	-0.7	-0.270	1.026	-3.707	0.778	-1.866	1.1693
Interest rates	2.8	3.022	2.542	1.729	1.089	-0.640	1.633
Assets	1.7	1.982	1.851	1.097	0.609	-0.332	0.907
SA dummy	1.8	0.971	2.141	2.921	2.015	2.676	1.292
Search costs (c)	1.3	1.257	0.976	-	-	0.073	1.213

[a] 20 simulations were performed
[b] 100 simulations were performed
[c] Has own customers
[d] No own customers

(BBsolve), *nleqslv (nleqslv)*, *SQUAREM (fpiter)* and *neldermead (fminsearch)* for solving the system of non-linear equations. *Nleqslv* solver uses the Newton method and the numerical gradients of the objective function; *BBsolve* is a derivative-free spectral approach for solving the systems of non-linear equations;[13] *fpiter* employs monotone, contraction mappings (including EM and MM algorithms); and *finminsearch* is a function that minimizes the objective function by using the Nelder-Mead algorithm. The inversion of the market shares with *finminsearch* achieved the tolerance criteria most often. Unfortunately this algorithm was very slow.[14] Therefore, we performed full model simulation and estimation procedures by using *BBsolve*. For the minimization of the GMM objective function we used *R* environment function *fminsearch*.[15] The estimation results are reported in Table 5.1 under the caption *Case I*.

Assumptions about how many alternatives bank customers may compare have a strong effect on the parameter estimates of the model. Thus, the estimated values of $\boldsymbol{\delta}$ and $\boldsymbol{\beta}$ depend on the assumptions about c and customer search behaviour. If

[13] Reynaerts et al. (2010) performed the inversion of the market shares of BLP model by using *BBsolve*, *nleqslv* and a BLP contraction mapping algorithm. They found that *BBsolve* was very successful in convergence, this algorithm was faster than the Contraction mapping procedure and faster than *nleqslv* if the analytical gradients were not provided.

[14] It took about 2-4 days for one inversion.

[15] We also tried standard *R* optimization routines *nlm* and *optim*, *BBoptim*. However, *fminsearch* converged more accurately.

the data is generated with costly consumer search but the model is estimated as if the search costs are zero then the values of δ are not equal to their true values and $\hat{\boldsymbol{\beta}}$ is biased. We simulated the market with costly consumer search and estimated $\boldsymbol{\beta}$ by setting $c = 0$. The summary of 100 simulations is written in Table 5.1 and is labeled *Case II*. The results show that the estimated values of $\boldsymbol{\beta}$ depart from their true values a lot if the search cost is ignored.

Bank customers become less picky if they have positive search costs. In other words, they accept worse bank reputation, lower interest rates than they would accept if the search costs were zero. As a result, the parameters in the utility function of a bank customer are underestimated if the search cost is set equal to zero in the estimation, while it is positive in reality. This result is clearly seen in our simulation results, where the interest rate coefficient is biased downwards.

Finally, we checked how the parameters of the demand function changed if the search costs were modeled as a characteristic of a contract instead of as a cost limiting consumers' consideration sets. In order to do this we generated the distance variable by drawing 10 values from $U(0, 7)$ for 10 banks. Then we assumed that all bank customers searched two banks at most (there were no customers with zero search costs and non-searching customers) and generated the market shares according to above described procedure. In addition, the search costs were specified as $(c + \nu) D_j$ on the LHS of (5.2) instead of $c + \nu$. The variable D_j indicated the distance to bank j. Afterwards we specified a different utility function for a bank customer who signed contract k function, i.e.

$$u_k = \mathbf{x}_k \boldsymbol{\beta} - (c + \nu) D_j + \xi_k + \varepsilon_k = \delta_k - \nu D_j + \varepsilon_k \tag{5.5}$$

where ν is bank and consumer specific term and $\nu \sim U(-1, 1)$. The model was estimated by assuming that bank customers had the utility specification (5.5) and observed all the existing utilities of all the banks. The estimation results are written in table 5.1 under the caption *Case III*. The estimated parameters are biased in this case too. Furthermore, the parameter next to the interest rates is negative, which points towards a misspecified model.

5.5 Discussion

"*The purpose of demand estimation is often to retrieve price elasticities and to calculate their effect on optimal pricing.*"[16] Thus, the specification of a demand function and its

[16] Davis and Garcés (2010), Chapter 9 "*Demand Estimation in Merger Analysis*".

estimated parameters have a strong impact on the assessment of a potential merger, the evaluation of new business regulations and investment. If the demand function is erroneously specified then its application for further calculations leads to incorrect results. Hence, it is important to explore the behaviour of consumers and their preferences for the correct specification of the demand function.

In this chapter we have presented a modification of the multinomial logit (MNL) demand function by assuming costly consumer search. Contrarily to the standard MNL assumption, in our model the choice set of a bank customer does not necessarily contain all the saving contracts that exist in the market. Limited choice sets arise due to positive search costs. If it is very costly for a customer to search a bank then the customer may not include the offers of this bank into her consideration set. Wrong assumptions about a consumer's consideration set lead to biased parameter estimates. Our simulation results show that the estimated parameters of the utility function are biased downwards if search costs are positive and ignored or modeled improperly in estimation.

The model estimation was run on simulated data. Furthermore, quite stringent restrictions were imposed on customer search behaviour. If the model is estimated with real data then the exact information about bank customer choices has to be incorporated and other model specifications (e.g. non-sequential search, sequential search with more than one search) should be explored. In addition, there are usually only a few banks in the retail banking market in a particular country. Therefore, the time dimension of the data set is longer than the cross-sectional dimension. Interest rates tend to follow ARMA processes, the expenditures of households are very likely to have some seasonal patterns. Thus, seasonal dummies and other explanatory variables may have to be incorporated in the model with the real data.

If consumers search sequentially and the search order is not known then the many-search-orders problem arises. With only a few banks one obtains rather many multidimensional integrals that do not have analytical expressions. This makes the estimation process very computationally intensive and slow. We had ten banks in one period in the simulated model. Hence, ten non-linear equations (5.2) had to be solved numerically for every draw of ν. As a result, the inversion of the market shares was very time consuming. Hence, some other estimation routines have to be explored.

Bayesian estimation procedures could be one of the options to make the estimation process faster and less sensitive to starting parameter values. It has been shown by Train (2009) that Bayesian procedures do not require to simulate the choice probabilities as many times as it is required for the maximization of the likelihood function. This option reduces the estimation time. Moreover, "*desirable estimation properties, such as consistency and efficiency, can be attained under more relaxed conditions*"[17]. However, in our model the market share expressions encompass the vector of unobservable product characteristics $\boldsymbol{\xi}$, which has to be tackled in the estimation process. Jiang et al. (2009) have proposed a method to estimate a mixed logit model with aggregated data and with vector $\boldsymbol{\xi}$ in the utility function. Their simulation results show that Bayes estimator has lower mean squared error than the standard GMM estimator. Additionally, the Bayesian estimation procedure does not require to use any instruments. However, this method still requires for an inversion of the market shares, which implies that solutions for δ have to be obtained for every iteration. Consequently, the problem of time consuming computational routines remains.

[17] Train (2009), Chapter 12 "*Bayesian procedures*".

Chapter 6

Conclusion

6.1 Summary of findings and policy implications

Assumptions about how consumers form their consideration sets and obtain the necessary information to ultimately conduct purchases have a strong impact on the strategic decisions of firms, including their decisions on price-setting, incentives to merge or collude, make long term investment, spend on advertising and promotion, etc. The decisions of competition authorities about the likelihood of misconduct of firms depend on the market modeling assumptions too. In this thesis, we have investigated the role played by positive consumer search costs on a number of strategic decisions by firms, including decisions on mergers, business reorganization and collusion. The main insight obtained is that ignoring that search costs can be significant in some markets may lead competition authorities to take the wrong decisions, thereby dampening economic activity and growth.

In a seminal contribution, Salant et al. (1983) showed that a seller that mergers with another firm earns less than in the pre-merger market when firms sell homogeneous products and compete by setting quantities. According to our analysis in Chapter 2, this so-called *merger paradox* also arises in a horizontally differentiated product market when firms compete by setting prices and consumer search costs are sufficiently high. Consequently, when consumers find it quite costly to search the market for satisfactory products there are no incentives to merge. If there are no incentives to merge then it is very unlikely that a fraction of all the firms choose to collude in the marketplace. Hence, if it is known that search costs are significant in a particular market, then competition authorities may spend less resources on market monitoring activities.

A merger is often approved if it results in merger specific cost efficiencies. The literature has focused on cost synergies or other forms of savings related to the production process. However, merger-specific efficiencies can also emerge at the demand side. It is shown in Chapter 3 of this thesis that firms that merge in a costly search market have incentives to dismantle their existing single-product firms and establish a multi-product store. In that case, if the search costs are sufficiently high, then consumers save on their total search costs and their post-merger surplus is higher than that in the pre-merger situation. Consequently, a merger is shown to be welfare improving even if it does not yield any production costs' efficiencies.

The economics literature has provided a good number of models in which consumers end up paying higher equilibrium prices as search costs increase. In Chapter 4 we show that there is one more threat for consumer surplus when the search costs are positive, namely, that a cartel is more stable if consumer search is more costly. This happens because high search costs make deviating from the cartel set-up less attractive, as less customers observe the deviation. Therefore, if authorities introduce and support initiatives to decrease consumer search costs, not only the competitive equilibrium prices decrease but also collusion becomes more complicated to sustain.[1]

As search costs increase, the average consideration set of a consumer shrinks. Hence, with positive search costs consumers are less choosy and accept products giving them lower utility than if search costs were negligible. As a result, estimation of demand models produces biased estimates if, as it is current practice to date, wrong assumption are made about search costs and search behavior. We show this in chapter 5, where we model the demand for saving deposits. Bank customers accept lower interest rates if the search costs for saving contracts are positive. Hence, provided that search costs are positive in reality, the interest rate coefficient is underestimated if instead it is assumed that search costs are equal to zero.

6.2 Directions for future research

Armstrong et al. (2009) and Zhou (2009) have shown that symmetric firms can charge different prices in equilibrium if consumers expect them to do so and respond by searching them in an optimal order. They have shown that firms earn

[1] For instance, the Central bank of Lithuania announces banking service fees of credit institutions on its web-site; Dutch consumers can find much information on financial products (or get directed to some comparison sites) on the web-site of AFM, can compare energy, telephone, internet contract offers on more than 10 comparison web-sites; Irish national consumer agency provides information about the contracts for banking, insurance services, etc.

more if they are prominent. Thus, sellers have incentives to pay for prominence, e.g. invest in advertising. However, symmetric firms have the same incentives to advertise. Hence, there is no equilibrium in pure advertising strategies and no seller would become prominent in the pre-merger market (see Haan and Moraga-González (2011)).

A merger introduces asymmetries between firms because of the internalization of pricing externalities. Hence, the incentives for firms to invest in prominence may differ across merged and non-merged firms. As a result, it would be interesting to explore the process of the formation of consumer expectations and how this process can be affected by e.g. advertising . In Chapter 2 of this thesis we have shown that there may be equilibria where the price of a merger is less than the price of the non-merged firms and, consequently, it is optimal for consumers to start their search for a satisfactory match at the merger. This equilibrium is driven by consumer expectations that are later confirmed. This equilibrium was valid just for some sets of parameter values. It would be worth to examine whether this equilibrium in the only equilibrium once we allow the firms to buy prominence in the marketplace.

A merger that yields search cost savings for consumers may be welfare improving, as shown in Chapter 3. In our modeling, however, we assumed that searching within the merged store is costless. This assumption may be unrealistic in situations where there are very many varieties being sold in the merged entity. When consumers must incur some smaller search cost to find the best variety within the merged entity, then the search costs and the expected benefit per variety in the merged entity depends on the search-history of a consumer. Hence, the myopic search rule (see Kohn and Shavell (1974) and Weitzman (1979)) is probably not valid any longer. Similarly, a consumer may get some information about not-yet-searched sellers when visiting other shops. Also in this case the expected benefits of an additional search would be search-history dependent. In future work, it may be interesting to look at history-dependent search processes and analyze them in more detail.

In Chapter 4 we have proposed a rather general saving deposit demand model but we have only analyzed in detail a simplified version of it. It may be the case that bank customers search more than one bank, or that the coefficients of the utility function are consumer specific. Therefore, it is worth to explore more sophisticated versions of the model, e.g. the random coefficients MNL. Another issue is that we did not estimate the model with real-world data, because of lack of them. Hence, applying the model to a particular market is a natural step we plan to take in the years to come. Finally, since the three stage optimization procedure we used in our chapter proves to be very time consuming, it would be interesting to explore other estimation methods for our model.

Samenvatting (Dutch Summary)

Een fusie tussen twee of meer bedrijven is doorgaans bevorderlijk voor de fusiepartners, maar schadelijk voor consumenten omdat hun surplus afneemt. Het klassieke model in Salant, Switzer, en Reynolds (1983) toont aan dat bedrijven nooit meer winst kunnen genereren als een fusie te klein is en de bedrijven met elkaar concurreren op hoeveelheden (à la Cournot). De situatie verandert echter als concurrentie op prijzen de norm is, a la Bertrand. In dat geval verdienen zowel de fusiepartners als hun concurrenten na de fusie meer dan voor het samengaan. Het consumentensurplus neemt altijd na de concentratie af, omdat de prijzen na de fusie toenemen. Dit negatieve effect voor consumenten kan minder worden als de productiekosten van fusiepartners na de concentratie afnemen, en de kwaliteit of de breedte van het assortiment van producten toenemen. Dit proefschrift draagt bij aan de economische literatuur door te analyseren wat de invloed is van zoekkosten ('*search costs*') op prikkels voor fusies, en hoe de fusie van invloed is op de welvaart van consumenten en producenten.

In een markt met productdifferentiatie en Bertrand competitie wordt de marktmacht van bedrijven sterker als de zoekkosten toenemen. Bovendien heeft de zoekvolgorde een invloed op prijzen en winsten van bedrijven. De winkels aan het begin van de zoekvolgorde vragen lagere prijzen dan de winkels aan het eind van de zoekvolgorde. Daardoor is het optimaal voor consumenten om in deze volgorde naar het product te zoeken waar de consument de meeste waarde aan hecht. Als er in deze markt een fusie plaatsvindt, zal er een verschil ontstaan tussen de prijs van fusiepartners en de prijs van hun concurrenten. Daarom verandert ook de zoekvolgorde in de (post-fusie) markt. In Hoofdstuk 1 nemen wij aan dat de fusie partners geen aanpassingen maken in hun assortiment of locatie, behalve dat ze prijzen gezamenlijk vaststellen. Er bestaat een marktevenwicht in ons model waar de prijs van de fusie partners is hoger dan de prijs van hun concurrenten. Daarom bezoeken consumenten eerst de concurrenten van de fusie voordat zij naar de winkels van de fusiepartners gaan. Consumenten zoeken minder als de zoekkosten stijgen. Daardoor verkopen de bedrijven aan het eind van de zoekvolgorde minder dan de anderen, bij een toename van de zoekkosten. Volgens onze analyse in Hoofdstuk 1 zullen, als de zoekkosten hoog zijn, de fusiepartners veel minder goederen verkopen dan ze voor de fusie verkochten. Daardoor kan de fusie paradox, die in de markt met homogene producten door Salant et al. gevonden is, ook in de markt met Bertrand competitie plaatsvinden.

Rationele consumenten zullen hun zoektocht beginnen bij de winkels met het hoogste verwachte consumentensurplus. Een winkel is aantrekkelijker als ze een lagere prijs kan bieden of de zoekkosten voor de consument verlaagt. Het is meestal niet optimaal voor de fusiepartners hun prijs lager dan de prijs van hun concurrenten te stellen. Daarom kan de fusie de zoekkosten proberen te verminderen. In Hoofdstuk 2 nemen wij aan dat de fusie partners al hun goederen in hun gezamenlijke winkel tentoonstellen. Zodoende kunnen consumenten voor dezelfde zoekkosten meerdere productvarianten vinden. Als zoekkosten hoog zijn, zullen consumenten dan ook bij de fusiewinkel beginnen te zoeken, ondanks dat de fusiepartners een hogere prijs vragen dan de concurrentie. De fusie leidt in dit geval tot een hogere winst voor de fuserende bedrijven, terwijl hun concurrenten minder verdienen dan voor de fusie. Bovendien wordt het consumentensurplus hoger door de fusie omdat op zoekkosten wordt bespaard. De totale welvaart na de fusie is hoger dan voor de fusie.

Zoekkosten hebben een invloed op de stabiliteit van geheime afspraken tussen bedrijven. Zonder zoekkosten hangt de stabiliteit van het kartel af hoe winstgevend het is af te wijken van de gemaakte afspraken. Hoofdstuk 3 toont aan dat deze winstgevendheid kleiner wordt als zoekkosten toenemen. De reden hiervoor is dat consumenten minder vaak zoeken, en minder consumenten de afwijking van de kartelafspraken opmerken. Bovendien zijn kartelafspraken tussen bedrijven onaantrekkelijker als de zoekkosten toenemen, omdat er minder concurrentie tussen de bedrijven is. Volgens ons resultaat neemt de kritieke verdisconteringsvoet waarboven het kartel niet stabiel is toe als de zoekkosten afnemen. Daarom is een kartel stabieler als de markt minder transparant is. Kortom, de kans op een kartel is hoger als de zoekkosten hoog zijn. Ons resultaat spreekt tegen het resultaat van Green en Porter (1984). Zij vonden dat een kartel minder stabiel is als de markt minder transparant wordt. Zij hebben de transparantie van een markt vanuit het standpunt van bedrijven onderzocht, terwijl onze analyse gaat om de transparantie van een markt vanuit het standpunt van consumenten.

Het aantal producten dat een consument vergelijkt voordat een aankoop wordt gedaan hangt af van de hoogte van zoekkosten. Daarom zijn de parameters van een berekend marktmodel van vraag en aanbod onjuist als de zoekkosten in de specificatie van het model genegeerd zijn. We bieden in Hoofdstuk 4 een methode aan om de zoekkosten in de markt met productdifferentiatie en multi-product bedrijven te berekenen. Het model is ontwikkeld voor de vraag naar spaardeposito's bij banken, en is zeer computerintensief. Desondanks kan deze methode ook in andere markten met een beperkt aantal ondernemingen toegepast worden.

References

Abreu, D., D. Pearce, and S. Ennio (1985). Optimal cartel equilibria with imperfect monitoring. *Journal of Economic Theory 39*, 251–269.

Albæk, S. and L. Lambertini (1998). Collusion in differentiated duopolies revisited. *Economics Letters 59*(3), 305–308.

Anderson, S. P. and R. Renault (1999). Pricing, product diversity, and search cost: a Bertrand-Chamberlin-Diamond model. *RAND Journal of Economics 30*(4), 719–735.

Arbatskaya, M. (2007). Ordered search. *RAND Journal of Economics 38*(1), 119–126.

Armstrong, M., J. Vickers, and J. Zhou (2009). Prominence and consumer search. *RAND Journal of Economics 40*(2), 209–233.

Bar-Isaac, H., G. Caruana, and V. Cuñat (2011). Search, design, and market structure. Unpublished manuscript.

Bexley, J. B. (2005). Why customers choose community banks: an empirical study. *Journal of Commercial Banking and Finance 4*, 81–89.

Boot, A. W. A. (2007). Competitiveness assessment and key issues for the Dutch retail banking sector. Report for OECD.

Boyd, W. L., L. Myron, and C. White (1994). Customer preferences for financial services: an analysis. *International Journal of Bank Marketing 12*, 9–15.

Brynjolfsson, E., A. A. Dick, and M. D. Smith (2010). A nearly perfect market. *Quantitative Marketing and Economics 8*, 1–33.

Burdett, K. and K. L. Judd (1983). Equilibrium price dispersion. *Econometrica 51*(4), 955–969.

Caplin, A. and B. Nalebuff (1991). Aggregation and imperfect competition: on the existence of equilibrium. *Econometrica 59*(1), 25–59.

Chang, M.-H. (1991). The effects of product differentiation on collusive pricing. *International Journal of Industrial Organization 9*, 453–469.

Competition Commission and the Office of Fair Trading (2010, September). Merger assessment guidelines. Joint publication with OFT.

Davis, P. and E. Garcés (2010). *Quantitative techniques for competition and antitrust analysis*. Princeton University Press.

De los Santos, B., A. Hortaçsu, and M. R. Wildenbeest (2011). Testing models of consumer search using data on web browsing and purchasing behavior. *American Economic Review*. forthcoming.

Deneckere, R. and C. Davidson (1985). Incentives to form coalitions with Bertrand competition. *The RAND Journal of Economics 16*(4), 473–486.

Deneckere, R. J. (1983). Duopoly supergames with product differentiation. *Economics Letters 11*(1-2), 37–42.

Devlin, J. and P. Gerrard (2005). A study of customer choice criteria for multiple bank users. *Journal of Retailing and Consumer Services 12*, 297–306.

Devlin, J. F. (2002). Customer knowledge and choice criteria in retail banking. *Journal of Strategic Marketing 10*, 273–290.

Devlin, J. F. and P. Gerrard (2004). Choice criteria in retail banking an analysis of trends. *Journal of Strategic Marketing 12*, 13–27.

DGCompetition (2010). Best practices for the submission of economic evidence and data collection in cases concerning the application of articles 101 and 102 TFEU and in merger cases.

Diamond, P. A. (1971). A model of price adjustment. *Journal of Economic Theory 3*(2), 156–168.

Dick, A. A. (2008). Demand estimation and consumer welfare in the banking industry. *Journal of Banking & Finance 32*, 1661–1676.

Draganska, M. and D. Klapper (2010). Choice set heterogeneity and the role of advertising: An analysis with micro and macro data. *Journal of Marketing Research 48*(4), 653–669.

Dubois, P. and H. Perrone (2010, February). Price dispersion and search costs: the roles of imperfect information and product differentiation. Unpublished manuscript.

Dudey, M. (1990). Competition by choice: the effect of consumer search on firm location decisions. *American Economic Review 80*, 1902–1105.

Ellison, G. and S. F. Ellison (2009). Search, obfuscation, and price elasticities on the internet. *Econometrica 77*(2), 427–452.

Ellison, G. and A. Wolitzky (2009, September). A search cost model of obfuscation. NBER Working Paper 15237.

European Central Bank (2010, September). EU banking structures.

European Commission (2007, June). Expert group on customer mobility in relation to bank accounts.

European Commission (2009). Tying and other potentially unfair commercial practices in the retail financial service sector.

European Commission (2010a, April). *EU Competition Law. Rules Applicable to Merger Control*. European Union. Competition handbook.

European Commission (2010b). Summary of responses to the public consultation on the study on tying and other potentially unfair commercial practices in the retail financial service sector.

Farrel, J. and C. Shapiro (1990). Horizontal mergers: An equilibrium analysis. *American Economic Review 80*(1), 107–126.

Federal Trade Commission (2010, August). Horizontal merger guidelines.

Fischer, J. H. and J. E. Harrington Jr (1996). Product variety and firm agglomeration. *RAND Journal of Economics 27*(2), 281–309.

Focarelli, D. and F. Panetta (2003). Are mergers beneficial to consumers? Evidence from the market of bank deposits. *American Economic Review 93*, 1152–1172.

Friedman, J. W. (1971). A non-cooperative equilibrium for supergames. *Review of Economic Studies 38*(1), 1–12.

Green, E. J. and R. H. Porter (1984). Non-cooperative collusion under imperfect price information. *Econometrica 52*(1), 87–100.

Haan, M. A. and J. L. Moraga-González (2011). Advertising for attention in a consumer search model. *Economic Journal 121*, 552–579.

Häckner, J. (1995). Endogenous product design in an infinitely repeated game. *International Journal of Industrial Organization 13*, 277–299.

Ho, K. and J. Ishii (2011). Location and competition in retail banking. *International Journal of Industrial Organization 29*, 537–546.

Hong, H. and M. Shum (2006). Using price distributions to estimate search costs. *RAND Journal of Economics 37*(2), 257–275.

Honka, E. (2010, September). Quantifying search and switching costs in the U.S. auto insurance industry. Unpublished manuscript.

Hortaçsu, A. and C. Syverson (2004). Product differentiation, search costs, and competition in the mutual fund industry: a case study of S&P 500 index funds. *Quarterly Journal of Economics 119*, 403–456.

Ishii, J. (2005, November). Compatibility, competition, and investment in network industries: ATM networks in the banking industry. Unpublished manuscript.

Ivaldi, M., B. Jullien, P. Rey, P. Seabright, and J. Tirole (2003a). The economics of Tacit collusion. IDEI Working Paper No. 186.

Ivaldi, M., B. Jullien, P. Rey, P. Seabright, and J. Tirole (2003b). The economics of unilateral effects. IDEI Working Paper No. 222.

Jacquemin, A. and M. E. Slade (1989). Cartels, collusion, and horizontal merger. Volume 1 of *Handbook of Industrial Organization*, pp. 415–473. Elsevier.

Janssen, M. C. and J. L. Moraga-González (2007, July). On mergers in consumer search markets. Tinbergen Institute Discussion Papers 07-054/1.

Janssen, M. C. W. and J. L. Moraga-González (2004). Strategic pricing, consumer search and the number of firms. *Review of Economic Studies 71*, 1089–1118.

Janssen, M. C. W., J. L. Moraga-González, and M. R. Wildenbeest (2005). Truly costly sequential search and oligopolistic pricing. *International Journal of Industrial Organization 23*, 451–466.

Jehiel, P. (1992). Product differentiation and price collusion. *International Journal of Industrial Organization 10*, 633–641.

Jiang, R., P. Manchanda, and P. E. Rossi (2009). Bayesian analysis of random coefficient logit models using aggregate data. *Journal of Econometrics 149*, 136–148.

Kim, E. H. and V. Singal (1993, June). Mergers and market power: evidence from the airline industry. *American Economic Review 83*(3), 549–69.

Kim, J. B., P. Albuquerque, and B. J. Bronnenberg (2010). Online demand under limited consumer search. *Marketing Science 29*, 1001–1023.

Klemperer, P. (1992). Equilibrium product lines: competing head-to-head may be less competitive. *American Economic Review 82*(4), 740–755.

Klemperer, P. and J. A. Padilla (1997). Do firms' product lines include too many varieties? *RAND Journal of Economics 28*(3), 472–488.

Knauff, M. (2006, February). Market transparency and Bertrand competition. CORE Discussion Paper No. 2006037.

Knittel, C. R. and V. Stango (2008). Incompatibility, product attributes and consumer welfare: evidence from ATMs. *Journal of Economic Analysis & Policy 8*.

Kohn, M. G. and S. Shavell (1974, October). The theory of search. *Journal of Economic Theory 9*(2), 93–123.

Koulayev, S. (2010, March). Estimating demand in online search markets with application to hotel bookings. Federal Reserve Bank of Boston Working Paper No. 09-16.

Lee, J. and J. Marlowe (2003). How consumers choose a financial institution: decision-making criteria and heuristics. *International Journal of Bank Marketing 21*, 53–71.

Lelieveldt, S. (2006). The Dutch interbank switching support service.

Lommerud, K. E. and L. Sørgard (1997). Merger and product range rivalry. *International Journal of Industrial Organization 16*(1), 21 – 42.

McAfee, R. P. and M. A. Williams (1992). Horizontal mergers and antitrust policy. *Journal of Industrial Economics 40*(2), 181–187.

Molnár, J., M. Nagy, and C. Horváth (2007). A structural empirical analysis of retail banking competition: the case of Hungary. MNB Working Papers No. 2007/1.

Moraga-González, J. L. and V. Petrikaitė (2011a). Consumer search costs and the incentives to merge under Bertrand competition. Tinbergen Institute Discussion Paper No. 11-099/1.

Moraga-González, J. L. and V. Petrikaitė (2011b). Horizontal mergers and economies of search. Unpublished manuscript.

Moraga-González, J. L., Z. Sándor, and M. R. Wildenbeest (2010). On the identification of the costs of simultaneous search. Indiana University, Kelley School of Business Working Paper No. 2010-10.

Moraga-González, J. L., Z. Sándor, and M. R. Wildenbeest (2011, February). Consumer search and prices in the automobile market. Unpublished manuscript.

Moraga-González, J. L. and M. R. Wildenbeest (2008). Maximum likelihood estimation of search costs. *European Economic Review 52*, 820–848.

Nakane, M., L. S. Alencar, and F. Kanczuk (2006, June). Demand for bank services and market power in Brazilian banking. Banco Central Do Brasil Working Paper No. 107.

Nevo, A. (2000). A practitioner's guide to estimation of random-coefficients logit models of demand. *Journal of Economics & Management Strategy 9*, 513–548.

Nevo, A. (2001). Measuring market power in the ready-to-eat cereal industry. *Econometrica 69*(2), 307–342.

Nilson, A. (1999, December). Transparency and competition. Stockholm School of Economics Working Paper No. 298.

Non, M. (2010, January). Joining forces to attract consumers: location choice in a consumer search model. MPRA Working Paper No. 20044.

OECD (2007, September). Review of competition in the Dutch retail banking sector.

Office of Fair Trading (2010, December). Advertising of prices.

Paswan, A. K., N. Spears, R. Hasty, and G. Ganesh (2004). Search quality in the financial services industry: a contingency perspective. *Journal of Services Marketing 18*, 324–338.

Perloff, J. M. and S. C. Salop (1985). Equilibrium with product differentiation. *Review of Economic Studies 52*(1), 107–120.

Perry, M. K. and R. H. Porter (1985). Oligopoly and the incentive for horizontal merger. *American Economic Review 75*(1), 219–227.

Porter, R. H. (1991). A review essay on Handbook of industrial organization. *Journal of Economic Literature 29*, 553–572.

Prager, R. A. and T. H. Hannan (1998). Do substantial horizontal mergers generate significant price effects? Evidence from the banking industry. *Journal of Industrial Economics 46*(4), 433–452.

Reynaerts, J., R. Varadhan, and J. C. Nash (2010). The convergence properties of the blp (1995) contraction mapping and alternative algorithms in R. Unpublished manuscript.

Robert, J. and D. O. Stahl II (1993). Informative price advertising in a sequential search model. *Econometrica 61*(3), 657–686.

Röller, L.-H. (2006). Economic analysis and competition policy enforcement in Europe. In *Modelling European Mergers: Theory, Competition Policy and Case Studies*. Edward Elgar Publishing.

Ross, T. W. (1992). Cartel stability and product differentiation. *International Journal of Industrial Organization 10*, 1–13.

Rothschild, R. (1997). Product differentiation and cartel stability: Chamberlin versus Hotelling. *Annals of Regional Science 31*, 259–271.

Salant, S. W., S. Switzer, and R. J. Reynolds (1983). Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium. *Quarterly Journal of Economics 98*(2), 185–199.

Schultz, C. (2005). Transparency on the consumer side and Tacit collusion. *European Economic Review 49*(2), 279–297.

Schultz, C. (2009, January). Collusion in markets with imperfect price information on both sides. CIE Discussion Paper No. 2009-01.

Schultz, N. and K. Stahl (1996). Do consumers search for the highest price? oligopoly equilibrium and monopoly optimum in differentiated-products market. *RAND Journal of Economics 27*(3), 542–562.

Shapiro, C. (1989). Theories of oligopoly behavior. Volume 1 of *Handbook of Industrial Organization*, pp. 329–414. Elsevier.

Stahl, D. O. (1989). Oligopolistic pricing with sequential consumer search. *American Economic Review 79*(4), 700–712.

Stahl, K. (1982). Differentiated products, consumer search, and locational oligopoly. *Journal of Industrial Economics 31*, 97–113.

Stigler, G. J. (1961). The economics of information. *The Journal of Political Economy 69*(3), 213–225.

Stigler, G. J. (1964). A theory of oligopoly. *The Journal of Political Economy 72*(1), 44–61.

Train, K. E. (2009). *Discrete choice methods with simulation*. Cambridge University press.

Varian, H. R. (1980). A model of sales. *American Economic Review 70*(4), 651–659.

Weitzman, M. L. (1979). Optimal search for the best alternative. *Econometrica 47*(3), 641–654.

Whinston, M. (2006). *Lectures on antitrust economics*. Cairoli lecture series. MIT Press.

Whinston, M. D. (2007). Antitrust policy toward horizontal mergers. Volume 3 of *Handbook of Industrial Organization*, pp. 2369–2440. Elsevier.

Wildenbeest, M. R. (2011). An empirical model of search with vertically differentiated products. *RAND Journal of Economics*. forthcoming.

Williamson, O. E. (1968). Economies as an antitrust defense: The welfare tradeoffs. *American Economic Review 58*(1), 18–36.

Wilson, C. M. (2010). Ordered search and equilibrium obfuscation. *International Journal of Industrial Organization 28*, 496–506.

Wolinsky, A. (1983). Retail trade concentration due to consumers'imperfect information. *Bell Journal of Economics 14*(1), 275–282.

Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. *Quarterly Journal of Economics 101*(3), 493–512.

Wolinsky, A. (2005). Procurement via sequential search. *Journal of Political Economy 113*(4), 785–810.

Zhou, J. (2009). Prominence and consumer search: the case with multiple prominent firms. MPRA Working Paper No. 12554.

Zhou, J. (2011). Ordered search in differentiated markets. *International Journal of Industrial Organization 29*(2), 253–262.

Zhou, X. (2008, December). Estimation of the impact of mergers in the banking industry with historical data on mergers. Unpublished manuscript.